Radar Reflectivity in Snowfall

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***Abstract*—Backscattering properties of dry snowflakes at different microwave frequencies are examined. It is shown that the Rayleigh approximation does not often provide the necessary accuracy for snowflake reflectivity calculations for radar wavelengths used in meteorology; however, another simple approximation, the Rayleigh–Gans approximation, can be safely used for such calculations. Reflectivity–snowfall rate relationships are derived for different snow densities and different radar frequencies. It is shown that dual-wavelength radar measurements can be used for estimating the effective sizes of snowflakes. Experimental data obtained during radar snowfall measurements in the WISP project of 1991 with the NOAA *X*- and *Ka*-band radars are found to be consistent with the described theoretical results.**

I. Introduction

Radar technologies have been widely used in many countries for studies of different types of precipitation [1]. Rainfall and snowfall measurements are the main meteorological interest in these studies. Usually, such measurements are based on the approximate relationships between the radar equivalent reflectivity factor Z_e (the principal radar observable usually measured in mm^6/m^3) and the precipitation rate R (in mm/h of water):

$$Z_e = AR^b. \tag{1}$$

Coefficients A and b depend on the precipitation type, and on the size and shape distributions and fall velocities of precipitation particles. A and b are not universal constants and show considerably greater variance for snowfalls than for rainfalls. The optimization of these coefficients in order to find an appropriate Z_e—R relationship has been an important problem in radar meteorology.

Many attempts have been made to obtain values of A and b for snowfalls. These values are usually found either theoretically (e.g., [2]–[5]) from analysis of snowflake size distribution spectra and corresponding calculations of radar reflectivities, or experimentally (e.g., [6]–[12]) from quasi-simultaneous measurements of radar reflectivities and ground data on snowfall rate or accumulation.

The first approach relies on the model presentation of snowflake spectra and the scattering mechanism. However, coefficients A and b in Z_e–R relationships found empirically for a particular experiment may not be representative of broader meteorological and geographical conditions and depend considerably on the sampling technique.

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In their classic works, Gunn and Marshall [2] and Sekhon and Srivastava [3], when they assumed applicability of the Rayleigh scattering mechanism and equivalence of scattering properties of snowflakes and corresponding melted water drops, found relatively large values for A (2000 and 1780, respectively) and b (2.0 and 2.21, respectively). Many other authors, however, experimentally found for dry snow much smaller values of A ranging from 230 [7] to 1050 [11]. Coefficient b is more stable and most authors give for it experimental values ranging from 1.6 to 2.0; however, the recent paper by Fujiyoshi *et al.* [9] gives $b = 1.09$.

Such a great variation of coefficients in the Z_e–R relationships, even for snowfalls with dry snowflakes, apparently reflects the natural variability of snowflake scattering properties. In this paper we shall theoretically analyze the dependence of snowfall reflectivities and Z_e–R relationships on snowflake size and density for different radar wavelengths, and investigate the applicability of the Rayleigh theory to describe radar backscattering in snowfalls. We shall also compare results obtained theoretically with radar measurements of snowfalls during the WISP (Winter Icing Storm Project) experiment of 1991.

II. General Theoretical Considerations

A. Backscattering Efficiency of a Snowflake

Snowflakes are usually complex aggregates of snow crystals with complicated shapes [13]–[14]. However, because of the lack of detailed studies of these shapes and the complexity of describing aggregate shapes, the spherical model for snowflakes is generally adopted now.

Backscattering properties of a spherical particle depend on the particle size and its complex refractive index ms. A snowflake can be considered a mixture of ice, water, and air [16], with P_i, P_w and P_a as their relative volume ratios:

$$P_i + P_w + P_a = 1 \tag{2}$$

A commonly used method to calculate the effective dielectric properties of mixtures is based on the Wiener's theory, which is discussed in [15]. According to this theory, the complex refractive index of snow can be calculated from the equation given in [16]:

$$(m_s^2 - 1)/(m_s^2 + u) = P_w(m_w^2 - 1)/(m_w^2 + u) + P_i(m_i^2 - 1)/(m_i^2 + u) \tag{3}$$

where m_i, m_w are complex refractive indices of water and ice, respectively, and dimensionless parameter u depends on the snowflake density. We took into consideration here that

TABLE I
COMPLEX REFRACTIVE INDICES OF DRY SNOW AT DIFFERENT MICROWAVE FREQUENCIES

	t = -10° C			t = -5° C		
ρ_s = 0.02 g/cm^3		0.04 g/cm^3	0.06 g/cm^3	0.02 g/cm^3	0.04 g/cm^3	0.06 g/cm^3
GHz band						
34. (Ka)	1.01404+ i0.000085	1.02869+ i0.000342	1.04397+ i0.000773	1.01404+ i0.000075	1.02872+ i0.000308	1.04405+ i0.000679
17. (Ku)	1.01406+ i0.000048	1.02879+ i0.000192	1.04420+ i0.000434	1.01406+ i0.000041	1.02880+ i0.000163	1.04422+ i0.000369
9.3 (X)	1.01407+ i0.000027	1.02882+ i0.000108	1.04426+ i0.000244	1.01407+ i0.000023	1.02882+ i0.000091	1.04427+ i0.000206
5.4 (C)	1.01407+ i0.000016	1.02883+ i0.000066	1.04428+ i0.000148	1.01407+ i0.000014	1.02883+ i0.000055	1.04428+ i0.000125
2.9 (S)	1.01408+ i0.000009	1.02884+ i0.000034	1.04429+ i0.000077	1.01408+ i0.000007	1.02884+ i0.000029	1.04429+ i0.000065

the refractive index of air is very close to unity ($m_a^2 \approx 1$), and we neglected the corresponding term in (3). Snow density ρ_s depends on the relative quantities of the ingredients in (2). It was empirically determined by Ihara *et al.* [16] that P_w and P_i are related to ρ_s as follows

$$\begin{aligned} P_w &= \rho_s^2 \\ P_i &= \rho_s(1-\rho_s)/\rho_i \end{aligned} \tag{4}$$

where ρ_i is the density of ice and both densities are in g/cm^3.

According to the data presented in [9] and [17], the density of falling dry snow usually is between 0.02 and 0.06 g/cm^3 and parameter u in (3) is about 2. However, in some cases ρ_s can be as small as 0.005–0.01 g/cm^3 [17]. Snow density values greater than 0.08 g/cm^3 correspond to moist and wet snow [16] for which parameter u varies from 8 to 20. Complex refractive index values of dry snow calculated for $-5°$C and $-10°$C using (3) and (4) are given in Table I. Those values are shown for microwave frequencies often used in radar meteorology. Data about m_i and m_w needed for calculations of m_s were taken from [18] and [17], respectively. It can be seen from Table I that real values of refractive indices of dry snow practically do not depend on the frequency. It is also true that

$$\mathrm{Re}(m_s) - 1 \gg \mathrm{Im}(m_s). \tag{5}$$

Temperature variations do not cause significant changes in $\mathrm{Re}(m_s)$. For example, a temperature decrease from $-5°$C to $-10°$C results in the changes of the difference $\mathrm{Re}(m_s) - 1$ of less than 1% for all considered frequencies and snow densities while $\mathrm{Im}(m_s)$ increases in this case approximately by 15–20%. However, the inequality (5) remains satisfied. All this means that the absorption of microwaves by dry snow is small in comparison with scattering, and the backscattering properties of a snowflake depend mainly on the snow density and the snowflake size factor $x = \pi D_s/\lambda$, where λ is the radar wavelength.

Usually, available data concerning snowflake sizes are given in terms of the diameters of the water drops to which the snowflakes would melt. In snowstorms the largest drop diameters of melted snowflakes (D) are about 6 mm [2], [13]. The real diameter of a snowflake (D_s) depends on the ratio of the snow (ρ_s) and water (ρ_w) densities

$$D_s = D(\rho_s/\rho_w)^{-1/3}. \tag{6}$$

One can easily estimate that the largest snowflakes can be as big as 2 cm for $\rho_s = 0.02$ g/cm^3. It is well beyond the size of applicability of the Rayleigh approximation especially for high frequencies. The conditions for the validity of this approximation, widely used for calculations of radar reflectivities in snowfalls, are

$$\begin{aligned} x &\ll 1 \\ |m_s|x &\ll 1. \end{aligned} \tag{7}$$

However, snowflakes are apparently within the limits of the validity for another rather simple approximation, the Rayleigh–Gans approximation. This approximation is usually used for calculating the scattering properties of particles with dielectric constants close to those of the medium (such particles are often called "soft"). The conditions for applicability of the Rayleigh–Gans approximation are the following [19]:

$$\begin{aligned} |m_s - 1| &\ll 1 \\ |m_s - 1|x &\ll 1. \end{aligned} \tag{8}$$

The Rayleigh–Gans approximation assumes that the electromagnetic field inside the particle can be approximated by the incident field, and each small volume of the particle produces the Rayleigh type scattering independently from other volumes. The equation for the backscattering cross section (σ) of a particle in this case is very simple [19]:

$$\sigma = (16\pi^3/\lambda^4)|m_s - 1|^2 V^2 f(x)^2 \tag{9}$$

where V is the particle volume and $f(x)$ is the shape factor. We note that (9) is valid for a particle of an arbitrary shape, and the cross section σ does not depend on the polarization of the incident electromagnetic wave, i.e., there is no polarization dependence of the reflectivities of "soft" particles even if they have very complex shapes. This explains the fact that radar reflectivities of dry snowfalls usually show very weak polarization dependence.

Generally, the shape factor $f(x)$ can be calculated for a particle of any shape. For a sphere, $f(x)$ is given by [19]:

$$f(x) = 3(\sin 2x - 2x\cos 2x)/(2x)^3. \tag{10}$$

For a spherical snowflake we can obtain the following equation for backscattering efficiency Q:

$$Q(x) = |m_s - 1|^2[\sin(2x)/2x - \cos(2x)]^2. \tag{11}$$

Fig. 1 shows the backscattering efficiency Q as a function of the size factor x for different snow densities. Q was calculated using the full Mie scattering theory and also in the Rayleigh and Rayleigh–Gans approximations. It can be seen from Fig. 1 that the Rayleigh approximation gives suitable results only for $x \leq 0.3$. Beyond this region discrepancies between results obtained from this approximation and from the Mie theory are larger than 5–6% for all the considered snow densities. The size factor $x = 0.3$ corresponds to the actual sizes of snowflakes of about 10 mm, 3 mm, 1.8 mm, and 0.9 mm for frequencies 2.9 GHz, 9.3 GHz, 17 GHz, and 34 GHz, respectively.

The Rayleigh–Gans approximation shows much better correspondence with the Mie theory data. The correspondence is better for small snow densities, because increasing the snow density results in an increased difference $|m_s - 1|$. The Rayleigh–Gans approximation reproduces the oscillating character of backscattering efficiency dependence on the size factor very well. It can be seen also from Fig. 1 that the correspondence is much better in the regions of maximal values of Q than in the minima. This fact is very important because snowflakes with high values of Q give the main contribution to the radar reflectivity Z_e.

B. Radar Reflectivity of an Ensemble of Snowflakes

Different observations of the snowflake spectra indicate that that the size distribution of aggregate snowflakes can be expressed by an exponential function in terms of melted diameters [2], [3], [13]:

$$N(D) = N_0 \exp(-\Lambda D). \tag{12}$$

In terms of snowflake actual diameters the size distribution can be rewritten

$$N_s(D_s) = N_{0s} \exp(-\Lambda_s D_s) \tag{13}$$

where $N_{0s} = N_0(\rho_s/\rho_w)^{1/3}$, and $\Lambda_s = \Lambda(\rho_s/\rho_w)^{1/3}$.

Ihara *et al.* [16] pointed out that exponential functions (12) and (13) describe size distributions of dry snowflakes and may not be appropriate for wet snowflakes. However, in this paper we are considering snowfalls with dry snow and will use these functions. In one of the first detailed studies of snowfall spectra

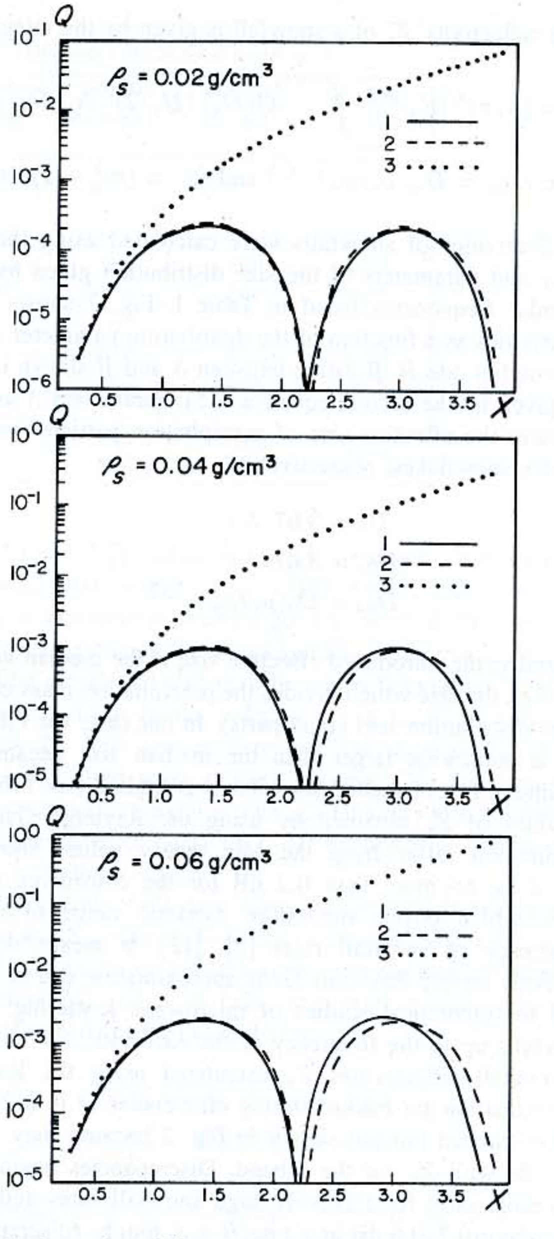


Fig. 1. Backscattering efficiency of a snowflake Q as a function of size parameter x for different snow densities: Predictions of the Mie theory (1), the Rayleigh–Gans approximation (2), and the Rayleigh approximation (3).

Gunn and Marshall [2] found for parameters of the exponential distribution

$$N_0(\mathrm{m^{-3}mm^{-1}}) = 3800R^{-0.87}$$
$$\Lambda(\mathrm{mm^{-1}}) = 2.55R^{-0.48} \tag{14}$$

where the snowfall rate is expressed in mm/h of melted snow. Later, Sekhon and Srivastava [3] made more precise estimations of these parameters

$$N_0(\mathrm{m^{-3}mm^{-1}}) = 2500R^{-0.94}$$
$$\Lambda(\mathrm{mm^{-1}}) = 2.29R^{-0.45} \tag{15}$$

and they found that the maximum melted diameter of snowflakes D_m in a spectrum depends on the parameter Λ

$$Dm = 6.4/\Lambda. \tag{16}$$

Radar reflectivity Z_e of a snowfall is given by the integral

$$Z_e = (\lambda/\pi)^4 \, |K_w|^{-2} \int_o^{D_{ms}} Q(D_s) \, (D_s/2)^2 \, N_s(D_s) \, dD_s \tag{17}$$

where $D_{ms} = D_m(\rho_s/\rho_w)^{-1/3}$ and $K_w = (m_w^2 - 1)/(m_w^2 + 2)$.

Reflectivities of snowfalls were calculated using the Mie theory and parameters of the size distribution given by (15) for radar frequencies listed in Table I. Fig. 2 shows these reflectivities as a function of the distribution parameter Λ and the snowfall rate R. Relation between Λ and R shown in Fig. 2 is given by the second equation (15). Parameters Λ and Λ_s represent the effective size of precipitation particles (melted and dry snowflakes, respectively):

$$\begin{aligned} D_0 &= 3.67/\Lambda \\ D_{0s} &= 3.67/\Lambda_s \\ D_{0s} &= D_0(\rho_s/\rho_w)^{-1/3}. \end{aligned} \tag{18}$$

Generally, the introduced effective size is the median volume size (i.e., the size which divides the precipitation mass content of the distribution into equal parts). In our case, the effective size is somewhat larger than the median size because the maximum size of melted snowflakes is limited (see (16)).

Values of Z_e obtained by using the Rayleigh–Gans approximation differ from the Mie theory values shown in Fig. 2 by no more than 0.2 dB for the considered region of R, which covers the entire dynamic range of natural occurrence of snowfall rates [2], [12]. It means that the relatively simple Rayleigh–Gans approximation can be safely used for theoretical studies of microwave scattering in dry snowfalls up to the frequency of the *Ka*-band.

Snowfall reflectivities Z_s calculated using the Rayleigh approximation for backscattering efficiencies Q in (17) were also calculated but not shown in Fig. 2 because they almost coincide with Z_e for the *S*-band. Discrepancies begin to be noticeable only for relatively high snowfall rates and reach only about 0.2–0.6 dB at a rate $R \approx 4$ mm/hr (discrepancies are greater for lower snow densities). Rayleigh reflectivities for equivalent water drops Z are also shown in Fig. 2. These reflectivities calculated from

$$Z = \int_o^{D_m} N(D) D^6 dD \tag{19}$$

have been often used for estimations of the snowfall reflectivities. It can be seen, however, that Z_e can differ from Z as much as several decibels.

Discrepancies between reflectivities in the *S*- and *X*- bands which are often used for the radar snowfall measurements are small for low R. For heavy snowfalls, however, these discrepancies can reach a few decibels. This conclusion is in an agreement with experimental data analyzed in [20]. These data show generally higher reflectivities in the *S*-band than in the *X*-band for the same snowfall rates. One can also see from Fig. 2 that the Rayleigh approach is not applicable to radar backscattering in the *Ka*-band for any snowfall rate. Reflectivities in this band increase with R at a relatively slow rate and usually are well below reflectivities in other bands used in radar meteorology.

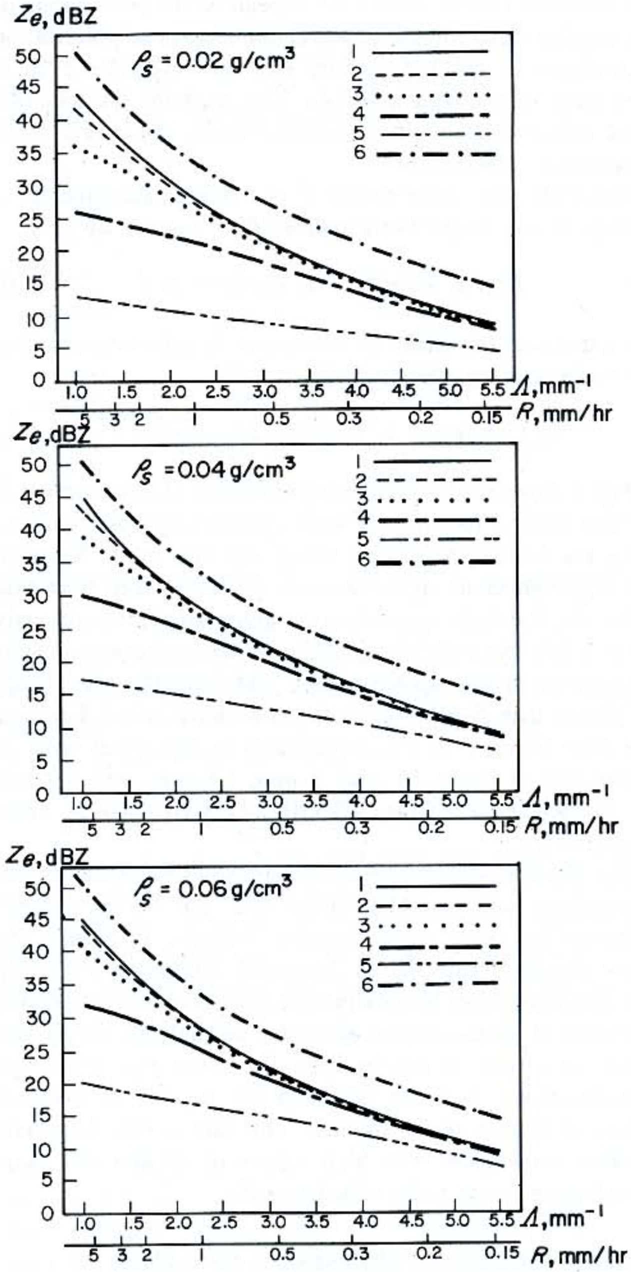


Fig. 2. Snowfall reflectivity (Z_e) dependence on snowfall rate (R) and size distribution parameter (Λ) for different snow densities and radar frequencies: (1) $\nu = 2.9$ GHz, (2) $\nu = 5.4$ GHz, (3) $\nu = 9.3$ GHz, (4) $\nu = 17.0$ GHz, (5) $\nu = 34.0$ GHz, (6) Rayleigh approximation for melted snowflakes.

III. Z_e—R Relationships for Snowfalls

Snowfall rate R in terms of equivalent water accumulation per unit of time can be inferred from the usual equation

$$R = (\pi/6) \, \rho_w \int_o^{D_m} v\,(D) \, D^3 N(D) \, dD \tag{20}$$

where $v(D)$ is the falling velocity of a snowflake which depends on the equivalent water drop diameter. The snowflake falling velocity dependence used in [2], [3] and many other

TABLE II
COEFFICIENTS IN Z_e—R RELATIONSHIPS FOR SNOWFALLS AT DIFFERENT MICROWAVE FREQUENCIES

		2.9 GHz S-band	5.4 GHz C-band	9.3 GHz X-band	17. GHz Ku-band	34. GHz Ka-band	Rayleigh dry snow	Rayleigh melted snow
snowflake density ρ_s g/cm^3								
0.02	A	870.	690.	410.	130.	10.0	950.	1950.
	b	2.01	1.90	1.60	1.00	0.50	2.03	2.22
0.04	A	570.	510.	340.	160.	20.0	610.	1950.
	b	2.01	1.95	1.75	1.20	0.61	2.03	2.22
0.06	A	460.	420.	240.	170.	28.0	490.	1950.
	b	2.02	1.98	1.95	1.35	0.95	2.03	2.22

investigations is [21]

$$v(D) = 2.07D^{0.31} \tag{21}$$

where v is in m/s and D is in cm. Later studies of falling snow indicated, however, that the falling velocities of snowflakes depend not only on their sizes (D_s or D) but also on the difference between densities of snowflakes (ρ_s) and the density of air (ρ_s) [22]:

$$v(D) = 8.8((\rho_s - \rho_a)D_s)^{0.5} \text{ (densities are in g/cm}^3\text{)}. \tag{22}$$

In our further analysis we used (22) rather than (21). It should be mentioned, however, that discrepancies between those equations can result in differences in the snowfall rate estimation as high as 30–40%.

Table II shows coefficients A and b in the Z_e—R relationships (see (1)) for snowfalls. These coefficients were found here theoretically for different wavelength regions and different snow densities. We also present data obtained under the Rayleigh assumption for dry and melted snowflakes. One can see that coefficients obtained for melted snowflakes ($A = 1950$, $b = 2.22$) are in good agreement with those found by Sekhon and Srivastava [3] ($A = 1780$, $b = 2.21$) and Gunn and Marshall [2] ($A = 2000$, $b = 2.0$). Values of b in the S- and C- bands are very close to 2, which does not differ significantly from b found for melted snowflakes. However, values of A are well below 1950. Both A and b decrease at the higher X- and Ku-band frequencies. In the Ka-band, radar reflectivities are relatively low due to strong non-Rayleigh effects and the oscillating dependence of backscattering cross sections of scatterers on their sizes.

The discussed results are in general agreement with the experimental studies cited in Section I. Authors of these studies usually found that coefficient A is often well below 2000 for dry snow and b is usually less than 2 in the X-band.

From Table II one can see also that Z_e–R relationships depend on the snow density. This dependency is small for b in the S- and C-bands but it is noticeable in the X- and Ku-bands. Contrary to this, coefficient A shows greater dependence on the snow density in the S- and C-bands in comparison with X- and Ku-bands. One of the reasons for the more variable A is the increase of snowflake falling velocities with increasing snow density (see (22)). Variations of A, however, do not exceed 3 dB when 0.02 g/cm^3 $\leq \rho_s \leq$ 0.06 g/cm^3, and if the snow density is unknown *a priori*, a good guess probably would be to use average values for A and b in the Z_e–R relationships. It should be mentioned also that the use of (21) rather than (22) for the snowflake falling velocity v can result in variations of A by 25–30% and even reverse the dependence of this coefficient on the snow density because (21) does not take into account variations of v with ρ_s. This fact also contributes to the inherent uncertainty of Z_e–R relationships due to snow density variations.

The frequently used snowflake size distribution functions of Sekhon and Srivastava (S–S) and Gunn and Marshall (G–M) imply that there is a one-to-one correspondence between parameters N_0 and Λ in (12). This correspondence can be easily obtained from (14) and (15)

$$\begin{aligned} N_0 &= 444\Lambda^{2.09} \; (S - S) \\ N_0 &= 700\Lambda^{1.81} \; (G - M). \end{aligned} \tag{23}$$

Although such correspondence between N_0 and Λ was found experimentally, there are some indications which do not confirm the existence of stable relationships between these two parameters. Braham [1990], in his study of snow particle size distributions in lake effect snowfalls, analyzed 49 different snowflake spectra. All these spectra could be satisfactorily described by the exponential function (12). We performed the statistical analysis of N_0 and Λ for this data set. This analysis showed the absence of significant correlation between these parameters. The correlation coefficient was only 0.14.

It is obvious that in situations with low correlation between N_0 and Λ (and hence between N_{0s} and Λ_s) it is convenient to have coefficient b close to 1. This coefficient is close to 1 when the reflectivity Z_e and the snowfall rate R depend on the snowflake effective size in a similar way. This results also in the fact that coefficient A depends mainly on parameter N_{0s} but not on Λ_s. A radar wavelength where $b \approx 1$ is the optimal one for snowfall measurements in the sense that the corresponding Z_e—R relationship does not depend on the scatterer effective size due to the non-Rayleigh effects. It may, however, not be the optimal wavelength in terms of sensitivity.

One can see from Table II that $b \approx 1$ at $\lambda = 1.8$ cm (*Ku*-band) for low snow densities. At the same time, as it is shown in Fig. 2, snowfalls produce noticeably less return signal at this wavelength in comparison with return signals in the *S*-, *C*-, and *X*-bands which are usually used for radar snowfall measurements. Nevertheless, wavelengths around $\lambda \approx 1.8$ cm could probably be used for snowfall studies when information about the structure of snowflake spectra is insufficient or doubtful.

It can be seen from Fig. 2 that snowfall reflectivities are monotone functions of the size parameter Λ_s (or the snowflake effective size D_{0s}). These functions, however, are different for different wavelengths. This suggests a means for estimating Λ from dual-wavelength radar measurements of snowfalls. The first wavelength could be one from the region with Rayleigh type scattering (*S*-band) or close to it (*C*-band or even *X*-band). The second wavelength could be either from the *Ku*- or from the *Ka*-band, where scattering is essentially non-Rayleigh but where Z_e still increases monotonically when Λ_s decreases. Radar reflectivities Z_e depend on both parameters of the snowflake size distribution (N_{0s} and Λ_s) while the logarithmic difference between Z_e at two such wavelengths depends only on Λ_s and is also a monotone function of this parameter.

Fig. 3 shows reflectivities in the *S*- and *X*- bands relative to reflectivities in the *Ka*-band as a function of the size distribution parameter Λ_s. The differences are greater than 2–3 dB for the whole region of natural occurrences of snowfall rates and hence can be measured with acceptable accuracy. These differences are also greater for snow with lower density since they primarily depend on snowflake actual sizes but not on melted sizes. For example, suppose that the difference between reflectivities at $\lambda = 3.2$ cm and $\lambda = 0.87$ cm is 4.8 dB. From Fig. 3 we can conclude that it corresponds to $Lambda \approx 3.7$ mm^{-1} and $D_0 \approx 1$ mm for $\rho_s = 0.06$ g/cm^3 and $\Lambda \approx 5.3$ mm^{-1} and $D_0 \approx 0.7$ mm for $\rho_s = 0.02$ g/cm3. Both of these effective sizes of melted snowflakes (D_0), however, correspond to the dry snowflake effective size $D_{0s} \approx 2.5$ mm (see (18)).

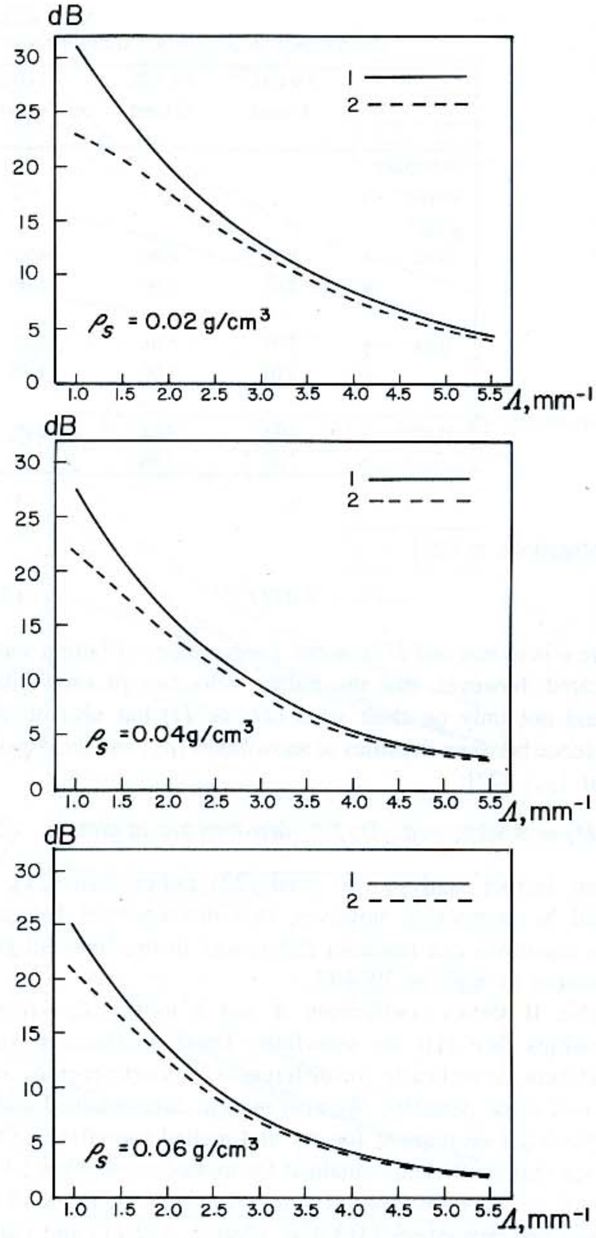


Fig. 3. Difference between reflectivities Z_e at two frequencies as a function on size distribution parameter Λ for different snow densities: (1) Z_e(2.9 GHz)–Z_e(34 GHz), (2) Z_e(9.3 GHz)–Z_e(34 GHz).

IV. Experimental Case of WISP 1991

During the Winter Icing and Storms Project (WISP) of 1991, the Wave Propagation Laboratory conducted combined dual-wavelength radar and microwave radiometer ($\lambda = 0.95$ cm and 1.45 cm) measurements of clouds and precipitation in northern Colorado. Radar measurements were performed with NOAA *X*-band ($\lambda = 3.22$ cm) and *Ka*-band ($\lambda = 0.87$ cm) parabolic antenna radars which have peak power 63 kW and 98 kW respectively. Synchronous low-elevation conical and RHI scans were performed with the colocated radars and radiometers whose beam centers were matched.

Fig. 4 shows a conical scan with 75-m range gate spacing at an elevation angle of 7.5° obtained at the Erie experimental site (about 20 km northeast of Boulder) on February 24, 1991 at 2039 UTC. This day was characterized by an extensive snowfall, which began approximately at 1600 UTC and almost stopped at 2400 UTC. There is a gap in the *Ka*-band radar data between azimuth angles 20° and 50°, because the *Ka*-band radar beam was blocked by the *X*-band radar at these azimuths.

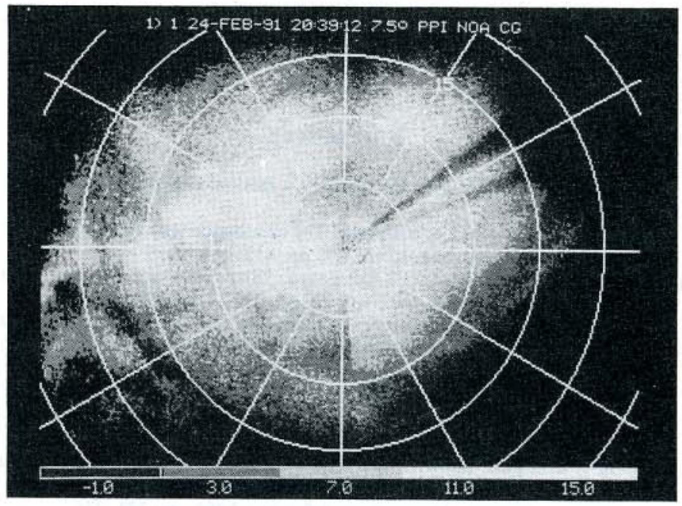
The site of ground snowfall rate and accumulation measurements was located at a range of 11.7 km in the azimuth direction of 260° from the radar site. According to the ground data, between 2000 and 2100 UTC, an average snowfall rate (R) in terms of equivalent water was about 0.2 mm/h. Average aggregate snowflake sizes were reported to be about 2–3 mm, and the snow depth–liquid equivalent depth ratio on the ground was about 13.

As one can see from Fig. 4, *X*-band reflectivities fairly well exceed those in the *Ka*-band. Both reflectivities were measured on the circular polarization. However, as mentioned above, polarization effects are very small for such "soft" scatterers as snowflakes and because of that there should not be any significant differences between the reflectivities on both linear (horizontal and vertical) and main circular polarizations. This conclusion was also confirmed by the experimental data. Received radar signals on the orthogonal circular polarization were negligibly small and corresponding circular depolarization ratios (CDR) during the discussed snowfall measurements were generally lower than the antenna polarization cancellation ratio (about -25 dB).

The mean values of reflectivity (Z_e) in the X- and Ka-bands for the radar resolution volume above the snow ground measurement site between 2000 and 2100 UTC were about 11.5 dBz and 4.5 dBz, respectively. These mean data were obtained by averaging the corresponding reflectivities for three conical scans performed between 2000 and 2100 UTC. Such a slow rate of scanning was dictated by the time required for microwave radiometers to achieve a suitable signal-to-noise ratio (SNR). These mean values are generally consistent with results from the Mie and the Rayleigh–Gans theories obtained above, which predict that the X-band reflectivity for dry snowfall with the melted equivalent precipitation rate $R \approx 0.2$ mm/hr should be somewhere between 9 and 15 dBZ and the difference between X- and Ka-band reflectivities should be from 3 to 8 dB, taking into consideration the uncertainty of the snowflake density.

Unfortunately, it is difficult to estimate the snowflake density ρ_s for more accurate comparisons of the theoretical and experimental data. Measurements of snow depth/liquid equivalent depth ratio on the ground provide the upper limit estimation of ρ_s because falling snow is less dense than snow on the ground. However, we can conclude that it was supposedly snowfall with dry snowflakes because liquid-water-sensitive radiometers did not show any significant amount of water and the upper limit estimation of the falling snow density from the mentioned above ratio gives $\rho_s < 0.075$ g/cm^3 which, according to [16], is less than typical values for moist snow.

From Fig. 3 and (18) we find that the experimentally obtained mean difference between Z_e in X and Ka-bands $\Delta Z_e \approx 7.0$ dB corresponds to the dry snowflake effective size $D_{0s} \approx 3.0$ mm, which is also consistent with the ground data. Estimations of this size do not show significant dependence on the snowflake density.



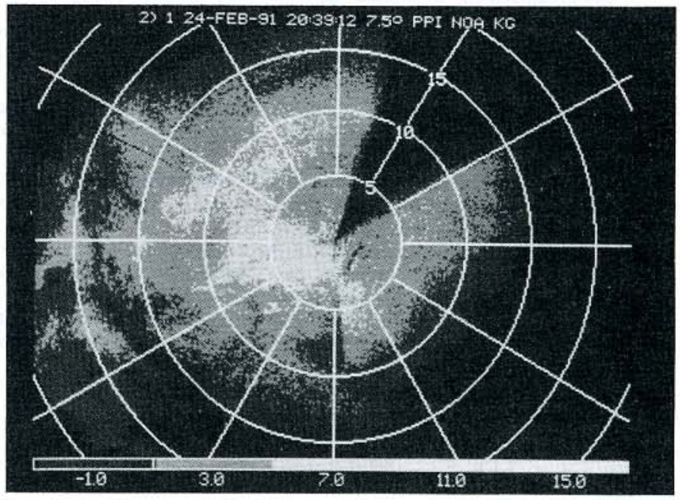


Fig. 4. Radar reflectivity fields in dBZ on 24 February 1991 at 2039 UTC during the WISP experiment; (1) X-band radar data, (2) Ka-band radar data. Range rings are 5 km from each other.

V. Conclusion

This paper contains a general theoretical study of radar backscattering of snowfalls. It shows that the conditions of the Rayleigh approximation are satisfied for dry snowflakes up to the size factor $x \approx 0.3$. Using this approximation for calculating radar reflectivities can significantly overestimate results for wavelengths $\lambda \leq 3$ cm. However, the relatively simple Rayleigh–Gans approximation gives satisfactory results for modelling radar observables for a large range of sensing frequencies up to the Ka-band and for the entire range of snowfall rate occurrences. This approximation also predicts a very low polarization dependence of radar signals reflected by dry snowflakes. It is also shown that Rayleigh reflectivities for melted snowflakes exceed those for dry snowflakes by a few decibels.

Because of non-Rayleigh scattering effects and the snow density variability, there is no universal Z_e—R relationship for snowfalls. Coefficients for this relationship were found theoretically for different densities of dry snow and different wavelengths used in radar meteorology. It is shown also that dual-wavelength radar measurements of snowfalls can provide information about the snowflake size distribution and their effective sizes. Simultaneous snowfall measurements with the X- and Ka-band radar during the WISP experiment of 1991 generally confirmed the theoretical results of this paper.

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