A New Nonhydrostatic Atmospheric Model Based on a Generalized Vertical Coordinate

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Outline

- Objective
- Governing equations
- Vertical transport of momentum (σ vs. θ)
- The vertical coordinate and diagnosis of the vertical velocity
- Results: 11 January 1972 Boulder, Colorado downslope windstorm (2D)
- Conclusions

Objective

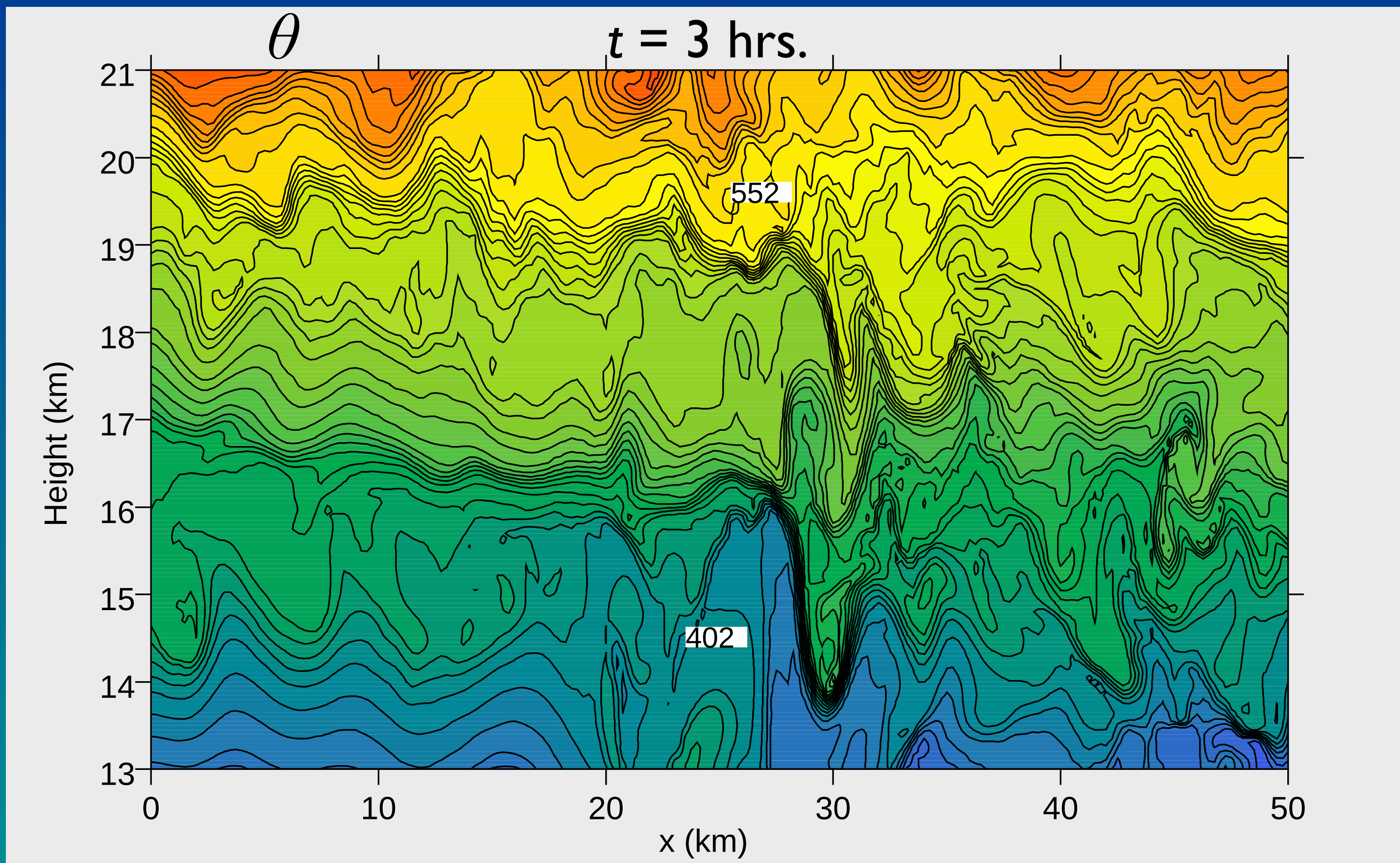
To model nonhydrostatic atmospheric motion, including isentropic overturning, with a quasi-Lagrangian vertical coordinate

On the small-scale...

Nonhydrostatic σ-coordinate model

$\Delta x = 0.25$ km

Breaking mountain wave



Nonhydrostatic governing equations in a generalized vertical coordinate (η)

- Horizontal momentum $$\frac{D\mathbf{v}}{Dt} + f\mathbf{k}\times\mathbf{v} = -\frac{1}{\rho}\nabla_\eta p + \frac{1}{m}\frac{\partial p}{\partial \eta}\nabla_\eta z + \mathbf{F}$$
- Vertical momentum $$\frac{Dw}{Dt} = -\frac{1}{m}\frac{\partial p}{\partial \eta} - g + F_z$$
- Mass continuity $$\left(\frac{\partial m}{\partial t}\right)_\eta + \nabla_\eta \cdot (m\mathbf{v}) + \frac{\partial}{\partial \eta}(m\dot{\eta}) = 0$$
- Thermodynamics $$\left(\frac{\partial \theta}{\partial t}\right)_\eta + \mathbf{v}\cdot\nabla_\eta \theta + \dot{\eta}\frac{\partial \theta}{\partial \eta} = \frac{Q}{\Pi}$$
- Height tendency $$\left(\frac{\partial z}{\partial t}\right)_\eta + \mathbf{v}\cdot\nabla_\eta z + \dot{\eta}\frac{\partial z}{\partial \eta} = w$$
- Pseudo-density $$m = \rho\frac{\partial z}{\partial \eta}$$
- Generalized vertical velocity $$\dot{\eta} = \frac{D\eta}{Dt}$$
- Material time derivative $$\frac{D}{Dt} = \left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)_\eta + \dot{\eta}\frac{\partial}{\partial \eta}$$

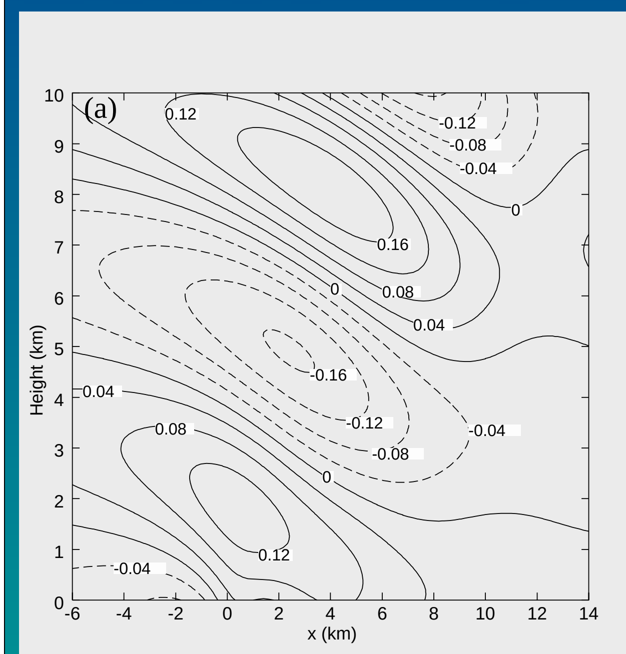
Simulation

Nonhydrostatic gravity waves in an isothermal, uniform flow over a small mountain

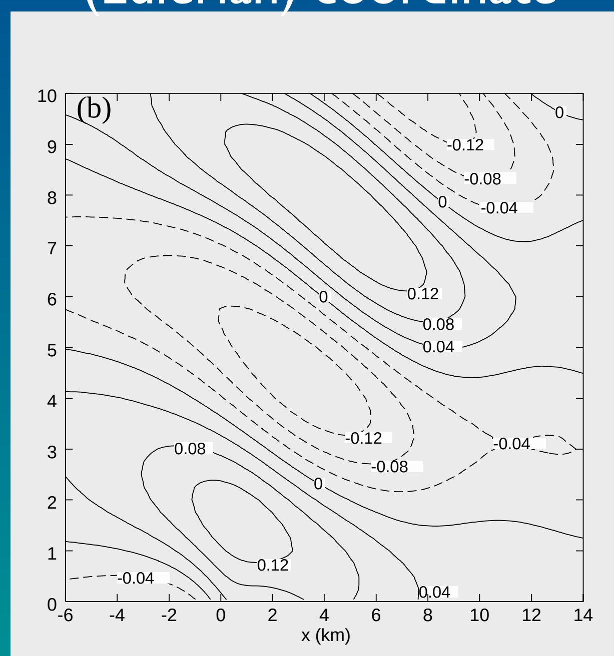
- Mountain height = 10 m
- Mountain half-width = 2 km
- Mean zonal wind = 20 m s^{-1}
- Steady state reached in ~ 1.11 hours

Perturbation zonal wind at t = 1.11 hours

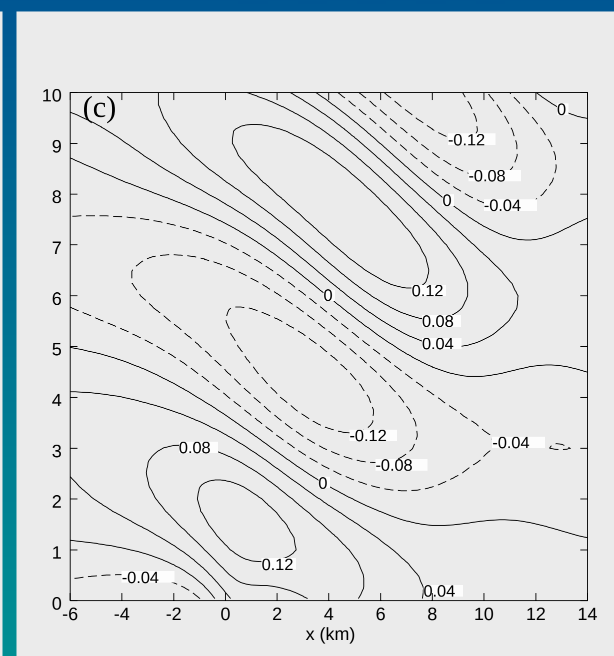
Analytical solution



Terrain-following σ (Eulerian) coordinate



θ coordinate (mostly)



Diagnosed momentum fluxes in a model experiment:

Nonhydrostatic gravity waves in an isothermal, uniform flow over a small mountain

$$\frac{\partial}{\partial t}\bar{u} = \frac{1}{\bar{m}}\frac{\partial}{\partial \eta}\left[\overline{p'\frac{\partial z'}{\partial x}} - \overline{(m\dot{\eta})'u'}\right] = 0$$

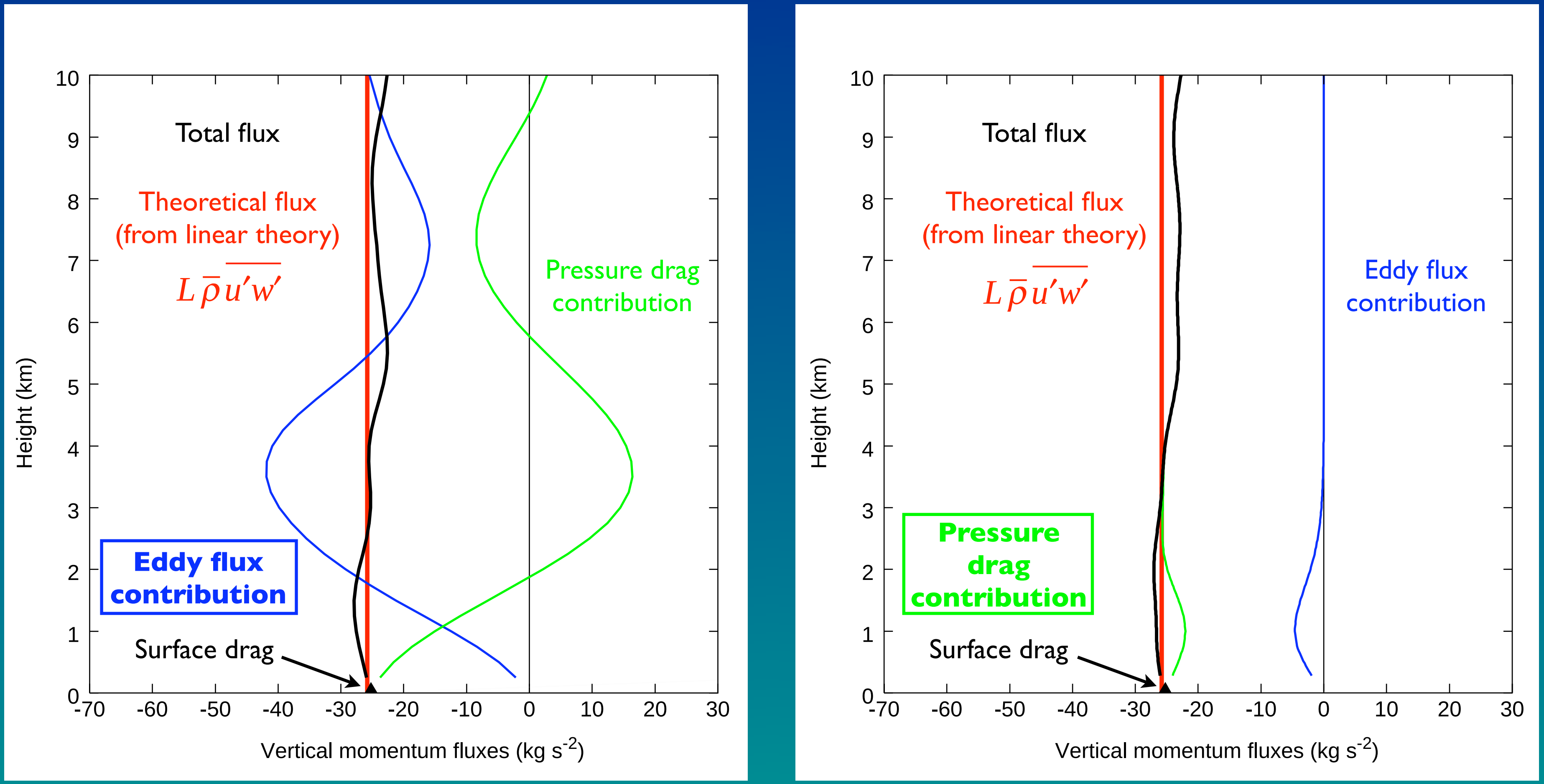
Vertical momentum flux
= constant

Small-amplitude gravity wave experiment

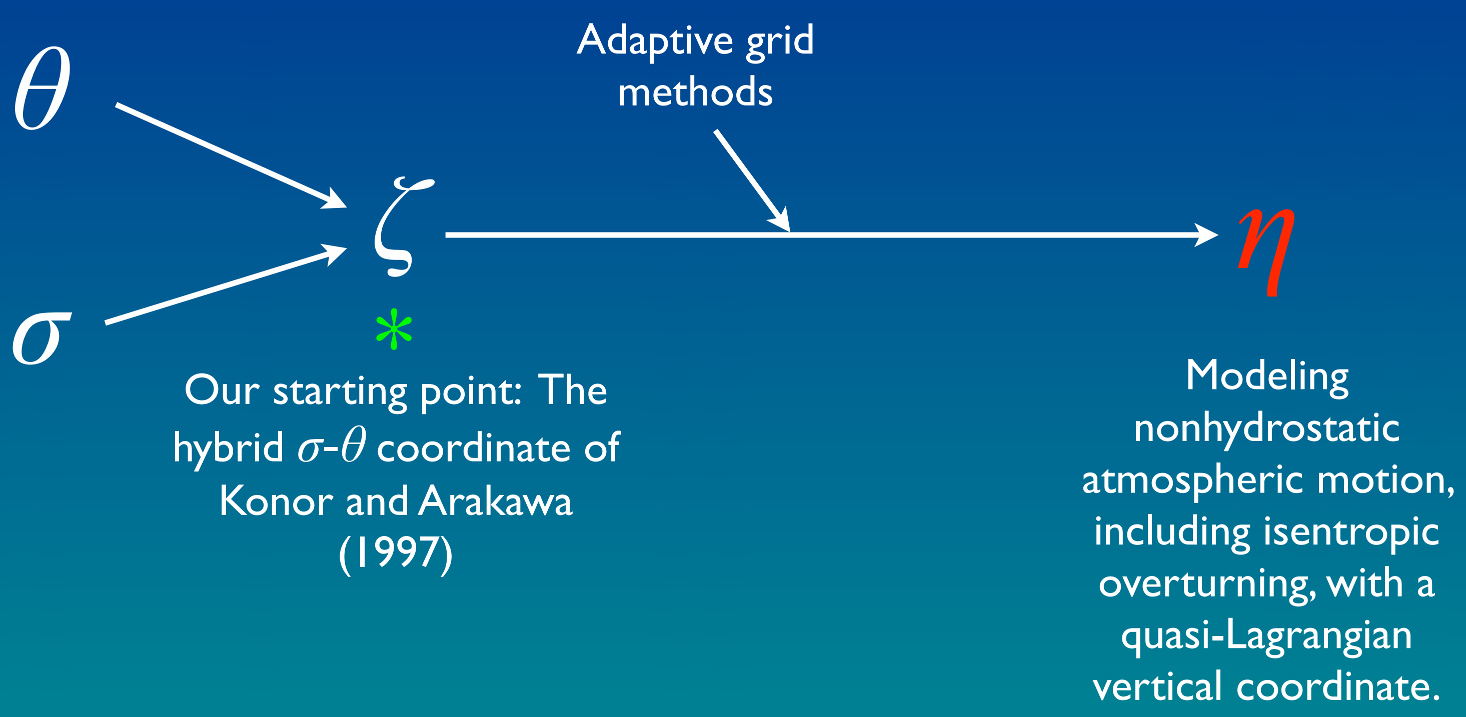
Profiles of vertical flux of horizontal momentum at t = 1.11 hours

Terrain-following σ (Eulerian) coordinate

θ coordinate (mostly)



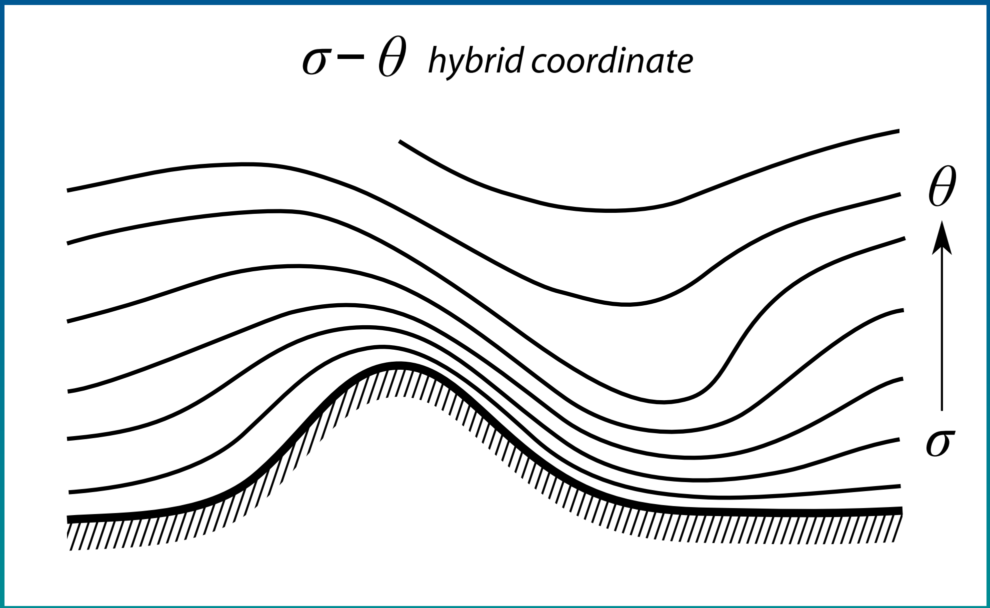
The design path to the η-coordinate model



Hybrid-coordinate method of Konor and Arakawa (1997) (KA97)

Generalized vertical coordinate (ζ)

Hybrid σ-θ: $\zeta \equiv F(\sigma,\theta) = f(\sigma) + g(\sigma)\theta,$



where terrain-following coordinate σ is defined by

$$\sigma \equiv \frac{z - z_S}{z_T - z_S} \begin{cases} = 0 & \text{at surface (S),} \\ = 1 & \text{at model top (T),} \end{cases}$$

and

$$\left.\begin{array}{lll} & g(\sigma) \to 0; & \sigma \to \sigma_S \\ f(\sigma) \to 0, & g(\sigma) \to 1; & \sigma \to \sigma_T \end{array}\right\}.$$

Diagnosis of KA97 vertical velocity ($\dot{\eta}_{KA97}$)

- On coordinate surfaces, $F(\theta,\sigma)$ is required to be constant:

$$\left(\frac{\partial}{\partial t}\right)_{\eta} F(\theta,\sigma) = 0$$

- Which leads to:

$$\dot{\eta}_{KA97} = \left(\frac{\partial F}{\partial \theta}\right)_{\sigma}\left(\frac{Q}{\Pi} - \mathbf{v}\cdot\nabla\theta\right) + \left(\frac{\partial F}{\partial \sigma}\right)_{\theta}\left(\frac{w - \mathbf{v}\cdot\nabla z}{z_T - z_S}\right)$$

Contribution from θ Contribution from σ

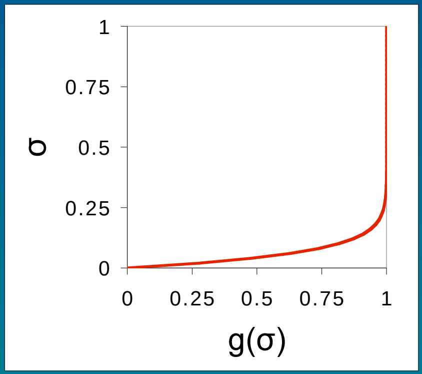
Hybrid-coordinate method of Konor and Arakawa (1997) (KA97)

Coordinate monotonicity requirement:

$$\frac{\partial \zeta}{\partial \sigma} > 0$$

$$\zeta \equiv F(\sigma,\theta) = f(\sigma) + g(\sigma)\theta$$

Determine $f(\sigma)$ and $g(\sigma)$ which satisfy:



$$\frac{df}{d\sigma} + \frac{dg}{d\sigma}\theta + g\frac{\partial \theta}{\partial \sigma} > 0$$

Choose: $g(\sigma) = 1 - (1-\sigma)^r$

Specify: $\frac{df}{d\sigma} + \frac{dg}{d\sigma}\theta_{\min} + g\left(\frac{\partial \theta}{\partial \sigma}\right)_{\min} = 0$

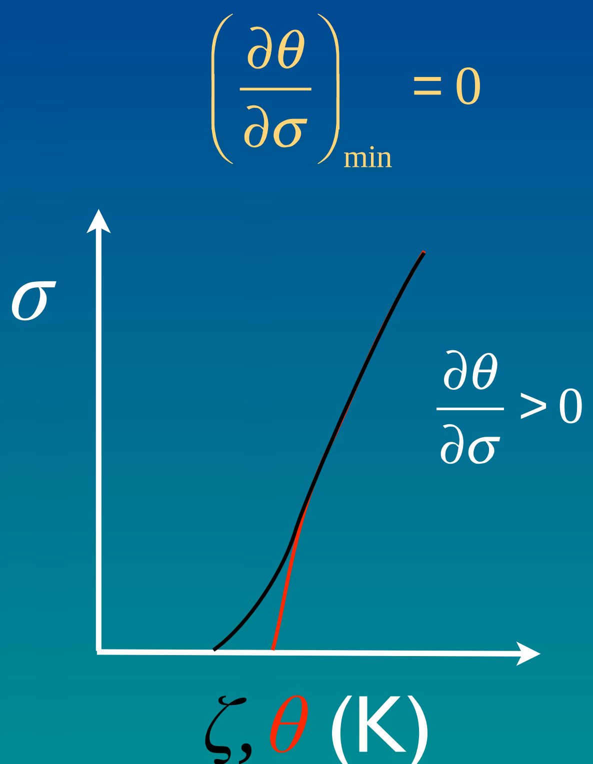
Suitably chosen parameters

Allows for negative static stability

Using $f(1)=0$, solve for $f(\sigma)$ and we're done.

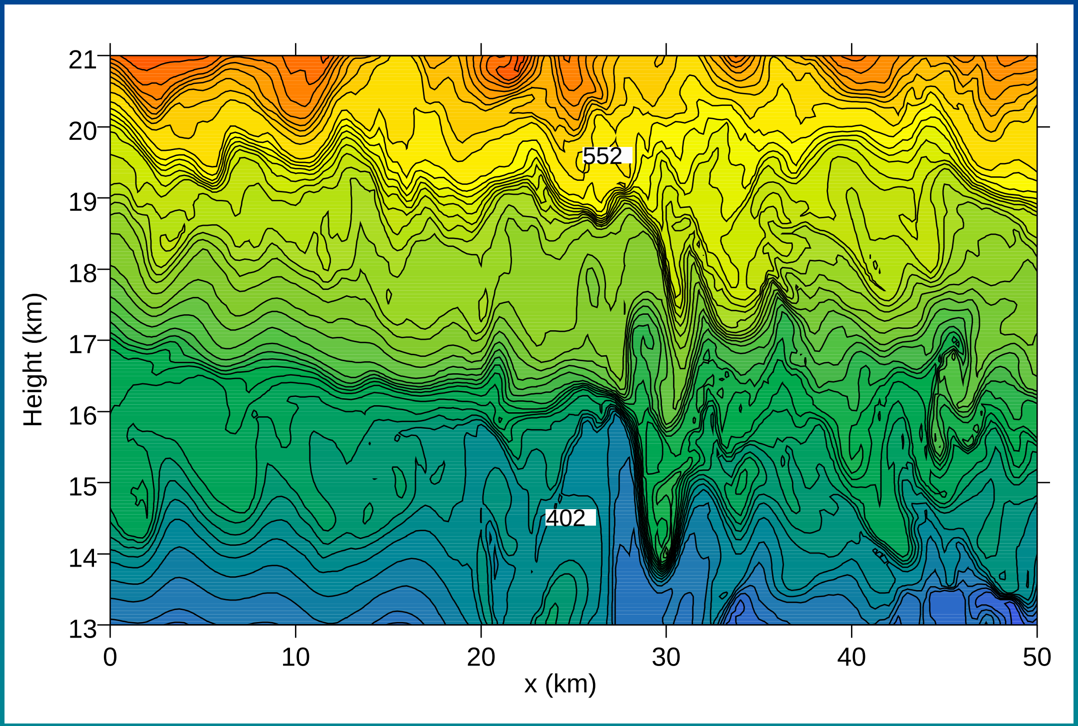
Hybrid-coordinate method of Konor and Arakawa (1997) (KA97)

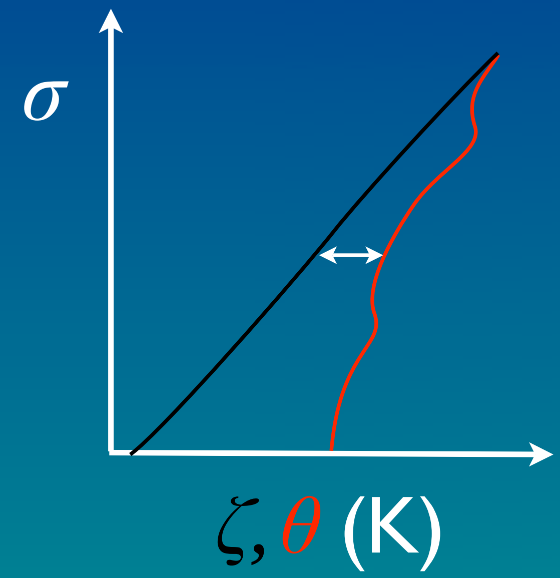
Effect of $\left(\frac{\partial\theta}{\partial\sigma}\right)_{\min}$ on ζ



Hybrid-coordinate method of Konor and Arakawa (1997) (KA97)

To represent this.... ⟶ $\left(\frac{\partial\theta}{\partial\sigma}\right)_{\min} << 0$

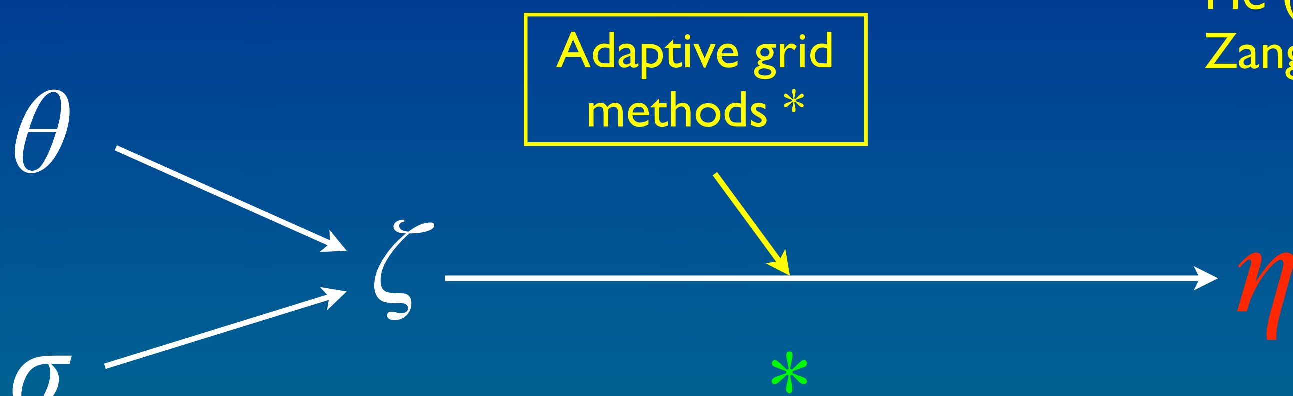




... σ has to dominate

Further along the design path to the η-coordinate model

* Skamarock (1998)
He (2002)
Zangl (2007)



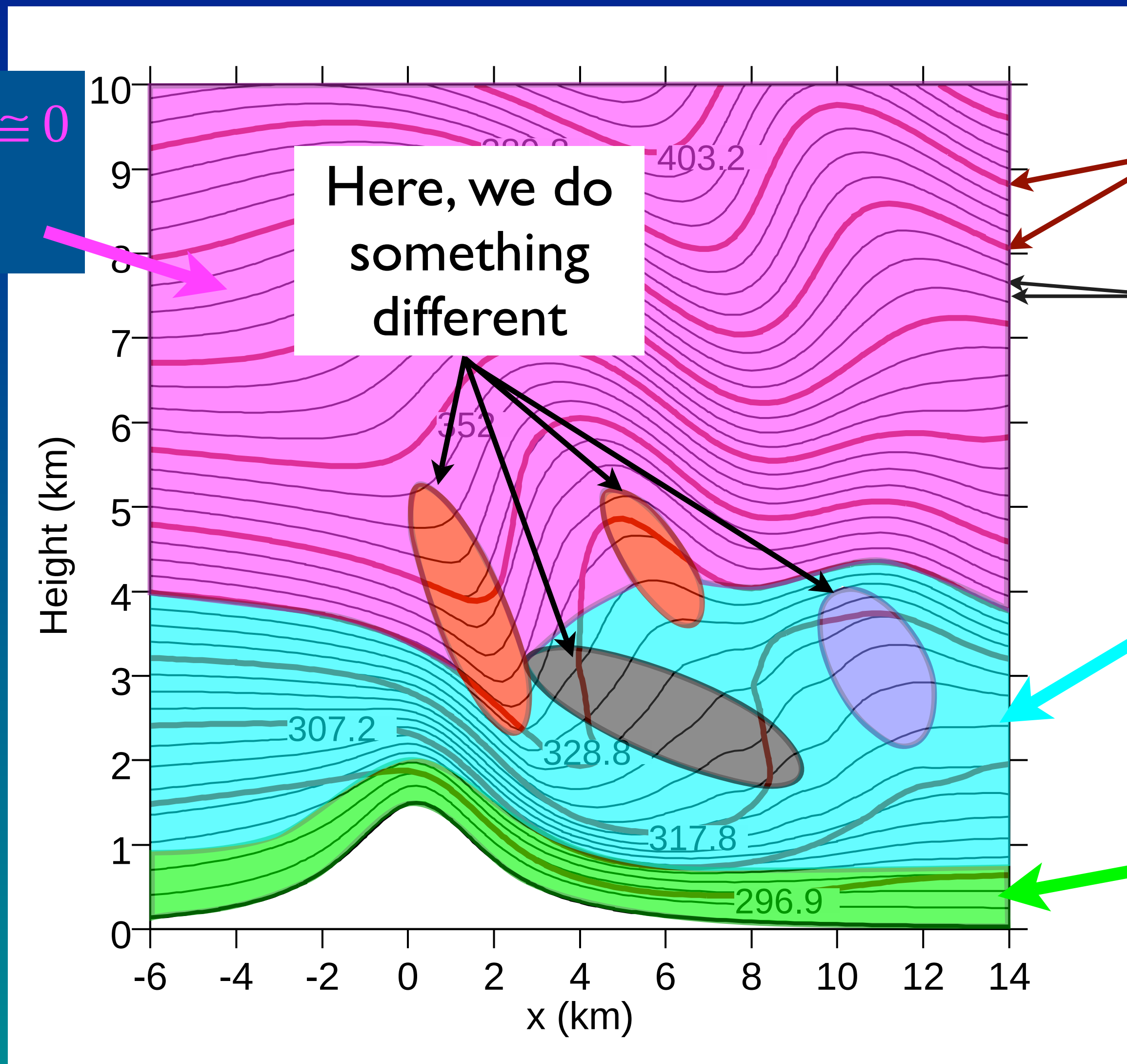
Our starting point: The hybrid σ-θ coordinate of Konor and Arakawa (1997)

Modeling nonhydrostatic atmospheric motion, including isentropic overturning, with a quasi-Lagrangian vertical coordinate.

$$\dot{\eta} = \dot{\eta}_{KA97} \cong \dot{\theta} \cong 0$$
$$\eta \cong \theta$$

isentropes (θ)

coordinate surfaces (η)



$$\dot{\eta} = \dot{\eta}_{KA97}$$
$$\eta = F(\theta, \sigma)$$

$$\sigma \equiv \frac{z - z_S}{z_T - z_S}$$

$$\dot{\eta} = \dot{\eta}_{KA97} \cong \dot{\sigma}$$
$$\eta \cong \sigma$$

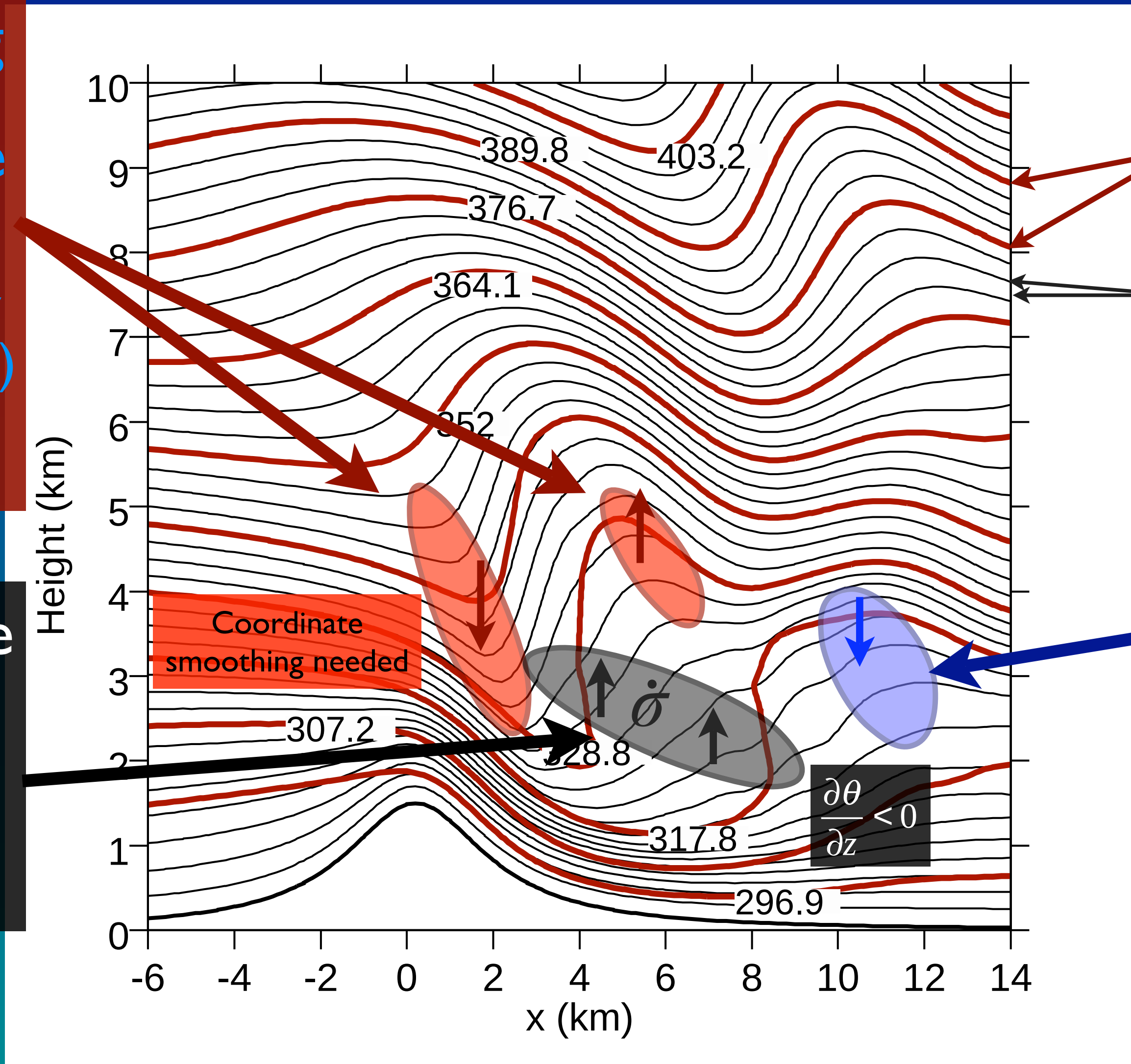
Adaptive grid

Smoothing the coordinate surfaces

(horizontally and vertically)

$\dot{\eta}_S$

Coordinate surfaces fixed in space: Eulerian



isentropes (θ)

coordinate surfaces (η)

Relaxation back to "target" isentrope

(An additional "target-seeking" component)

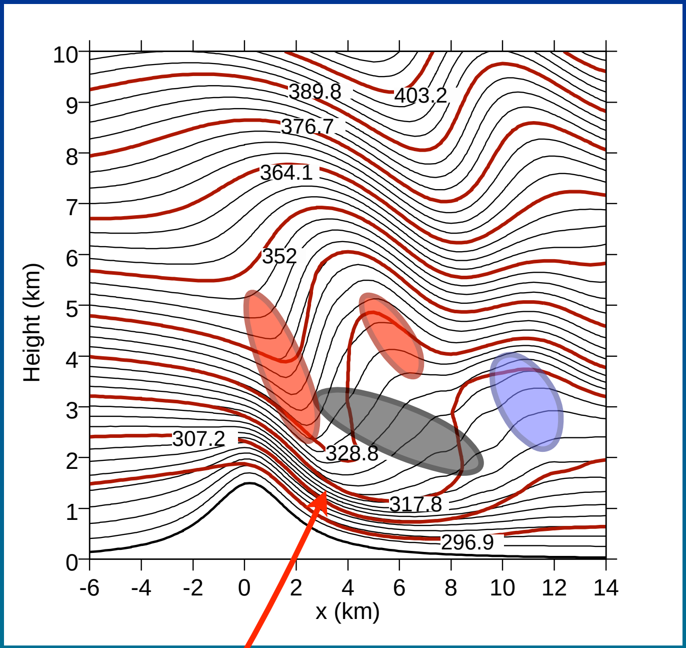
"Adaptive coordinate" modification to KA97

On coordinate surfaces, we require:

$$\left(\frac{\partial}{\partial t}\right)_{\eta} F(\theta,\sigma) = \frac{\eta - F(\theta,\sigma)}{\tau} - \dot{\eta}_s \frac{\partial F}{\partial \eta}$$

Relaxation of $F(\theta,\sigma)$ back to target η

Coordinate smoothing term



But not in these places

"Adaptive coordinate" modification to KA97

The two components of the generalized vertical velocity:

$$\dot{\eta} = \dot{\eta}_T + \dot{\eta}_S$$

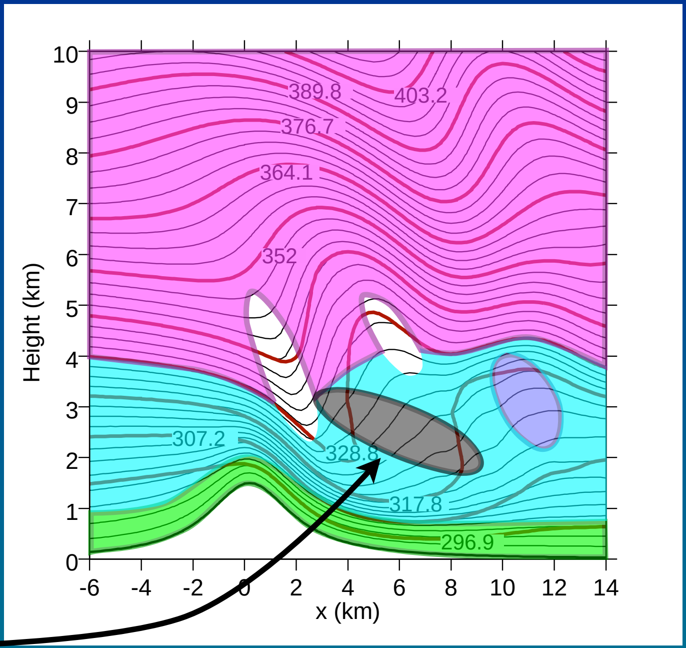
"Target-seeking" component

"Smoothing" component

$$\left(\frac{\partial}{\partial t}\right)_\eta F(\theta,\sigma) = \frac{\eta - F(\theta,\sigma)}{\tau} - \dot{\eta}_S \frac{\partial F}{\partial \eta}$$

Determining $\dot{\eta}_T$

$$\dot{\eta}_T = \left(\frac{\partial F}{\partial \eta}\right)^{-1} \left[\dot{\eta}_{KA97} + \frac{F(\theta,\sigma) - \eta}{\tau} \right]$$



In statically unstable regions:

$$\dot{\eta}_T = \left(\frac{\partial z}{\partial \eta}\right)^{-1} \left(w - \mathbf{v} \cdot \nabla z\right) \qquad \text{for } \frac{\partial F}{\partial \eta} \le 0$$

which fixes coordinate surfaces in space, i.e., $\dfrac{\partial z}{\partial t} = 0$

Determining $\dot{\eta}_S$

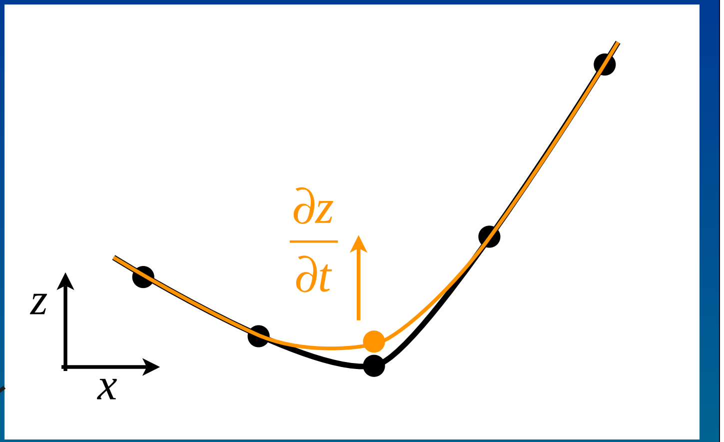
("Smoothing" component)

- maintains coordinate monotonicity
- smoothing is conditional

$$\dot{\eta}_S = -\left[\left(\frac{\partial z}{\partial t}\right)_{\text{smoothing}, h} + \left(\frac{\partial z}{\partial t}\right)_{\text{smoothing}, v}\right]\frac{\partial \eta}{\partial z}$$

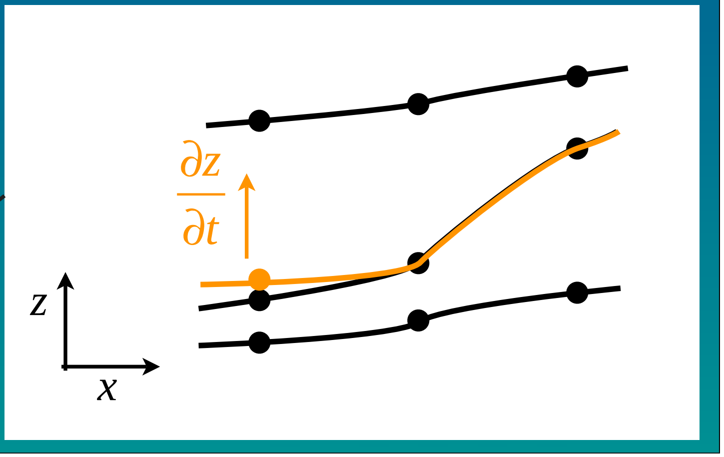
- Horizontal smoothing

$$\left(\frac{\partial z}{\partial t}\right)_{\text{smoothing}, h} \propto \text{diffusion term proportional to } \nabla^4 z$$



- Vertical smoothing

$$\left(\frac{\partial z}{\partial t}\right)_{\text{smoothing}, v} \propto \text{diffusion term proportional to } \partial^2 z/\partial \eta^2$$



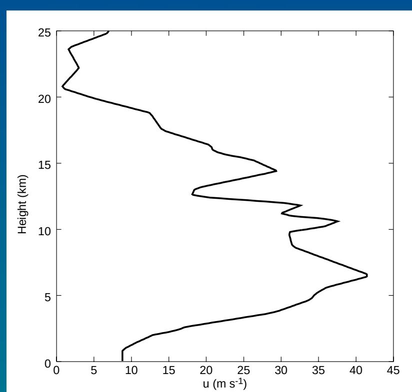
Simulations:
11 January 1972 Boulder, Colorado
Downslope Windstorm

- Vertical redistribution of zonal momentum through amplifying/breaking mountain waves
- A much studied case due to extensive observational data

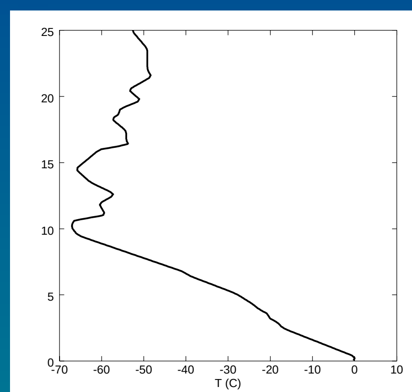
11 January 1972 Boulder, Colorado Downslope Windstorm

Initial conditions

- Based on upstream Grand Junction, CO sounding for 1200 UTC 11 January, 1972



Zonal wind component (m s^{-1})

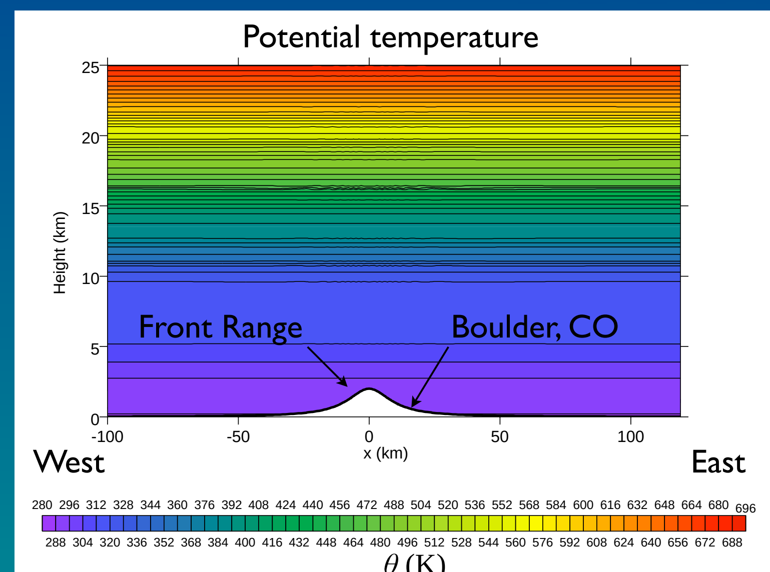


Temperature (˚C)

Experimental setup follows Doyle et al. (2000)

Model configuration

- 2D
- z_{TOP} = 48 km
- 205 levels
- 125 levels in the lowest 25 km
- average Δz = 200 m

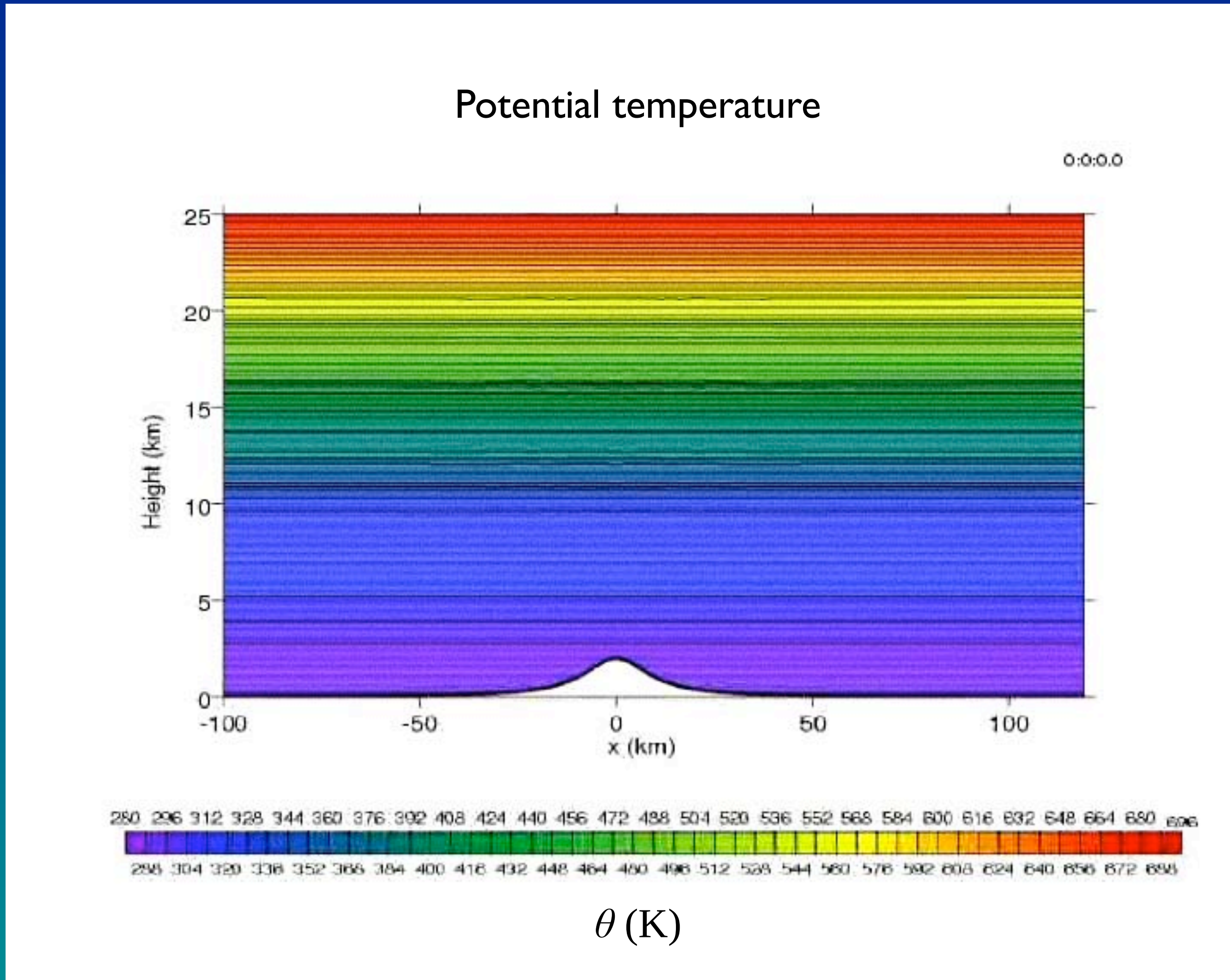


- Δx = 1 km
- Periodic horizontal domain
- Mountain height = 2 km

11 January 1972 Boulder windstorm simulations

σ-coordinate model run

"SIGMA125"

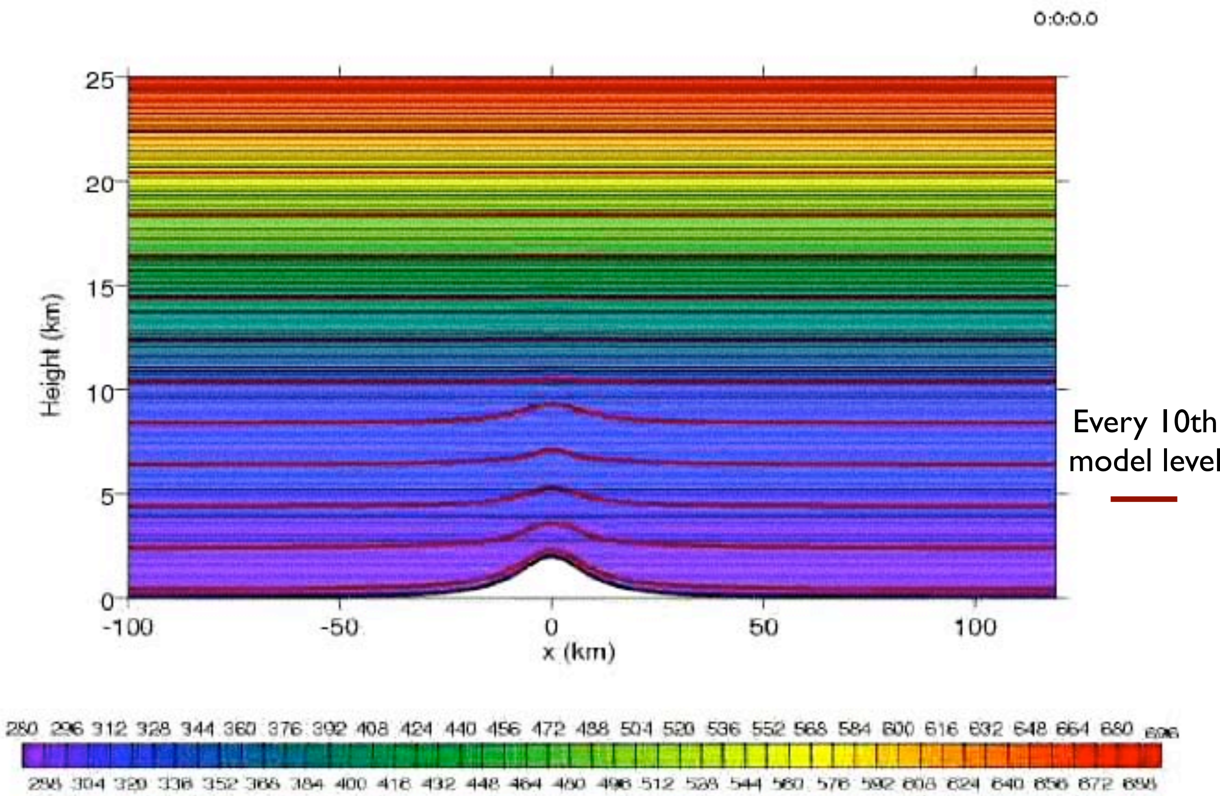


11 January 1972 Boulder windstorm simulations

Hybrid-coordinate model run

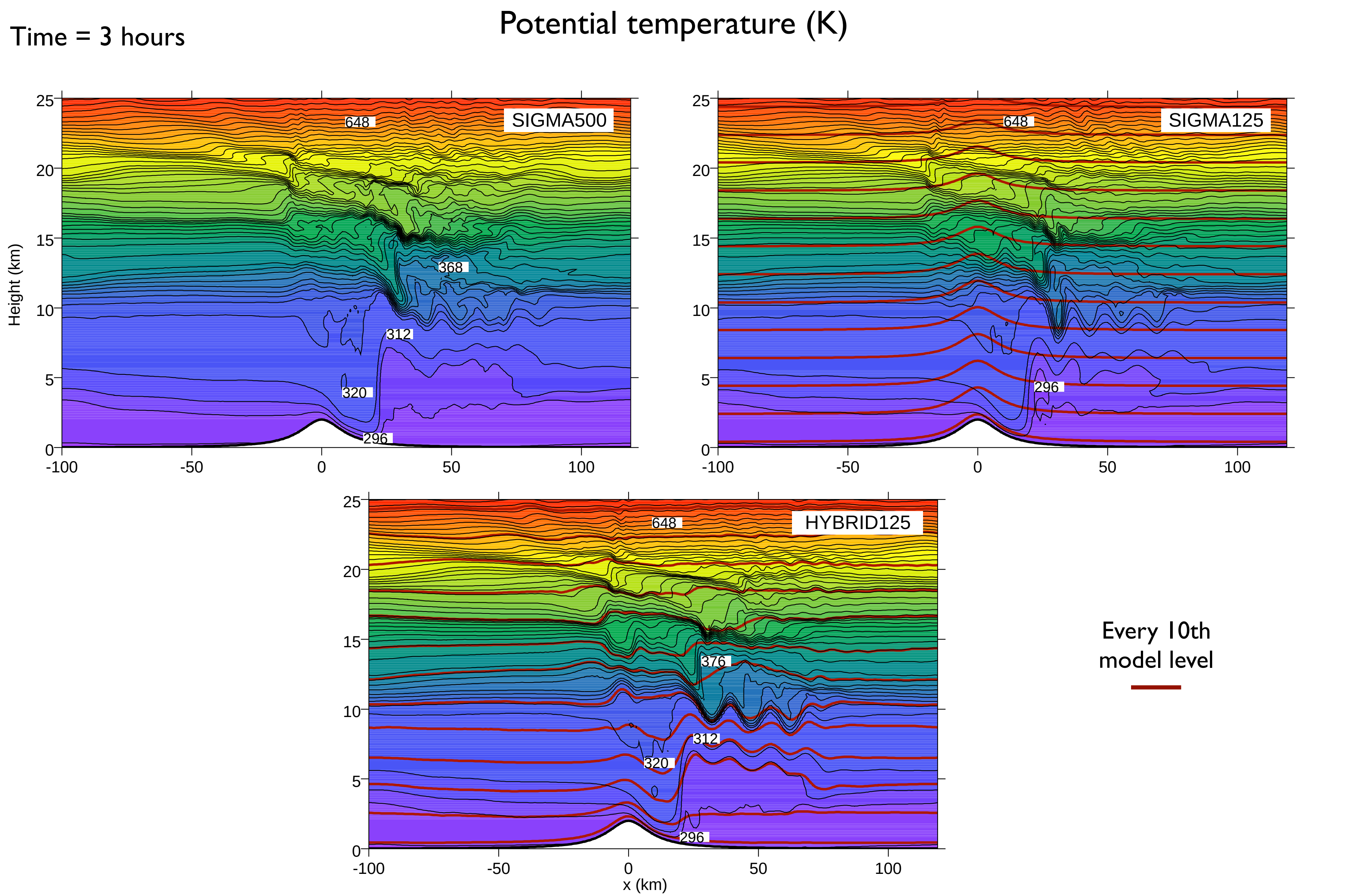
“HYBRID125”

Potential temperature



θ (K)

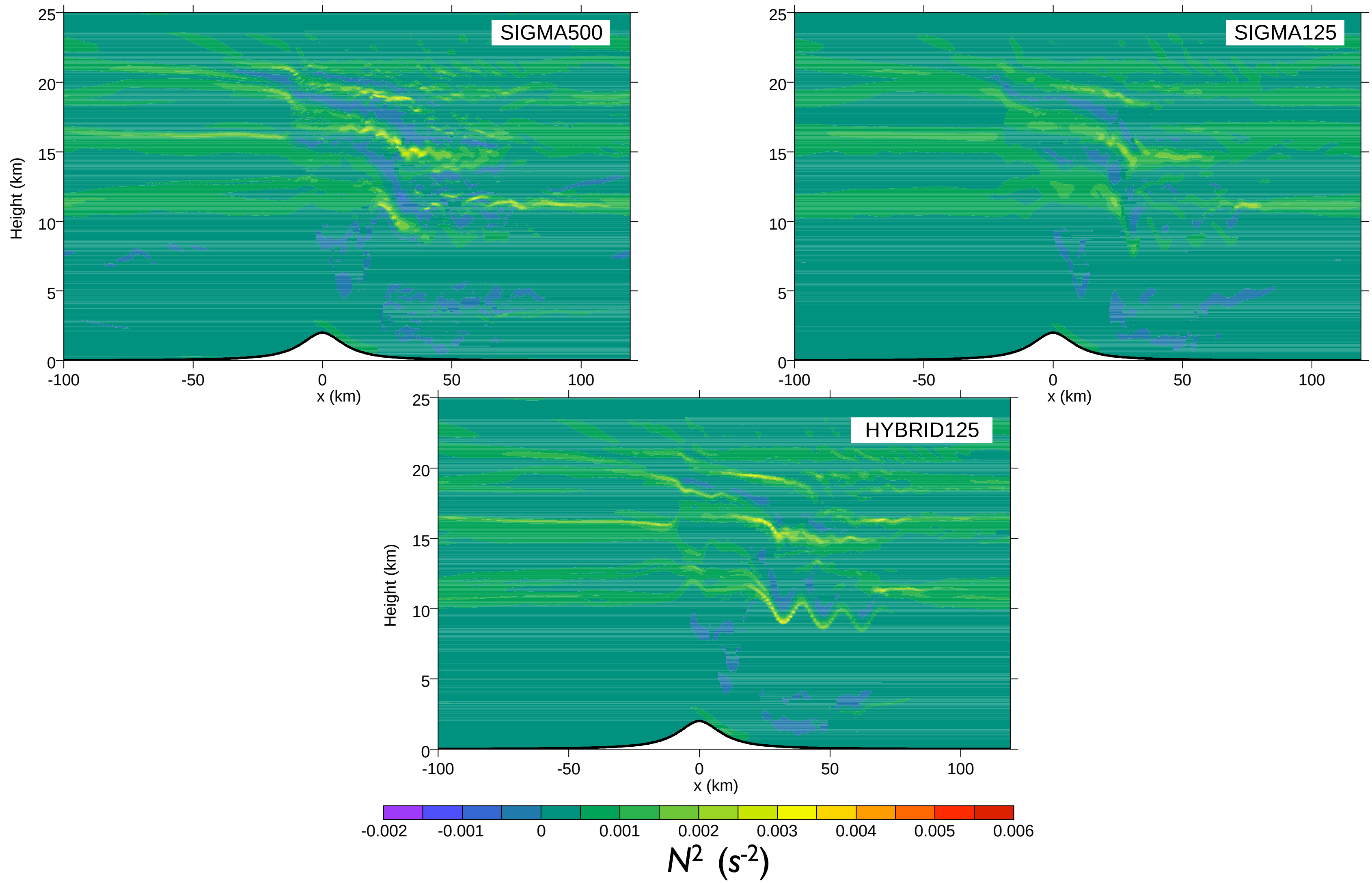
11 January 1972 Boulder windstorm simulations



11 January 1972 Boulder windstorm simulations

Time = 3 hours

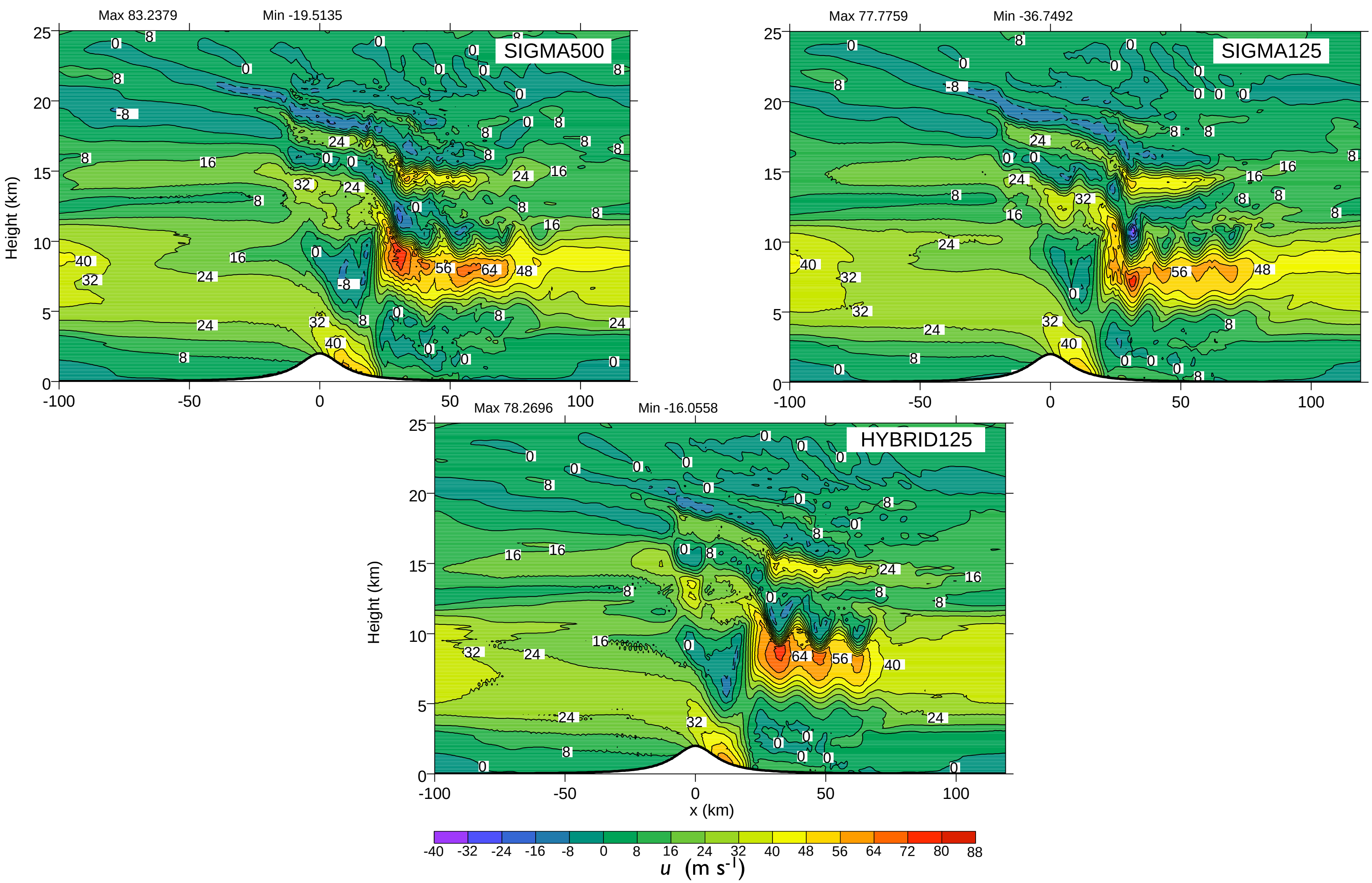
Static stability -- $N^2 = g\theta^{-1}\delta\theta/\delta z$



11 January 1972 Boulder windstorm simulations

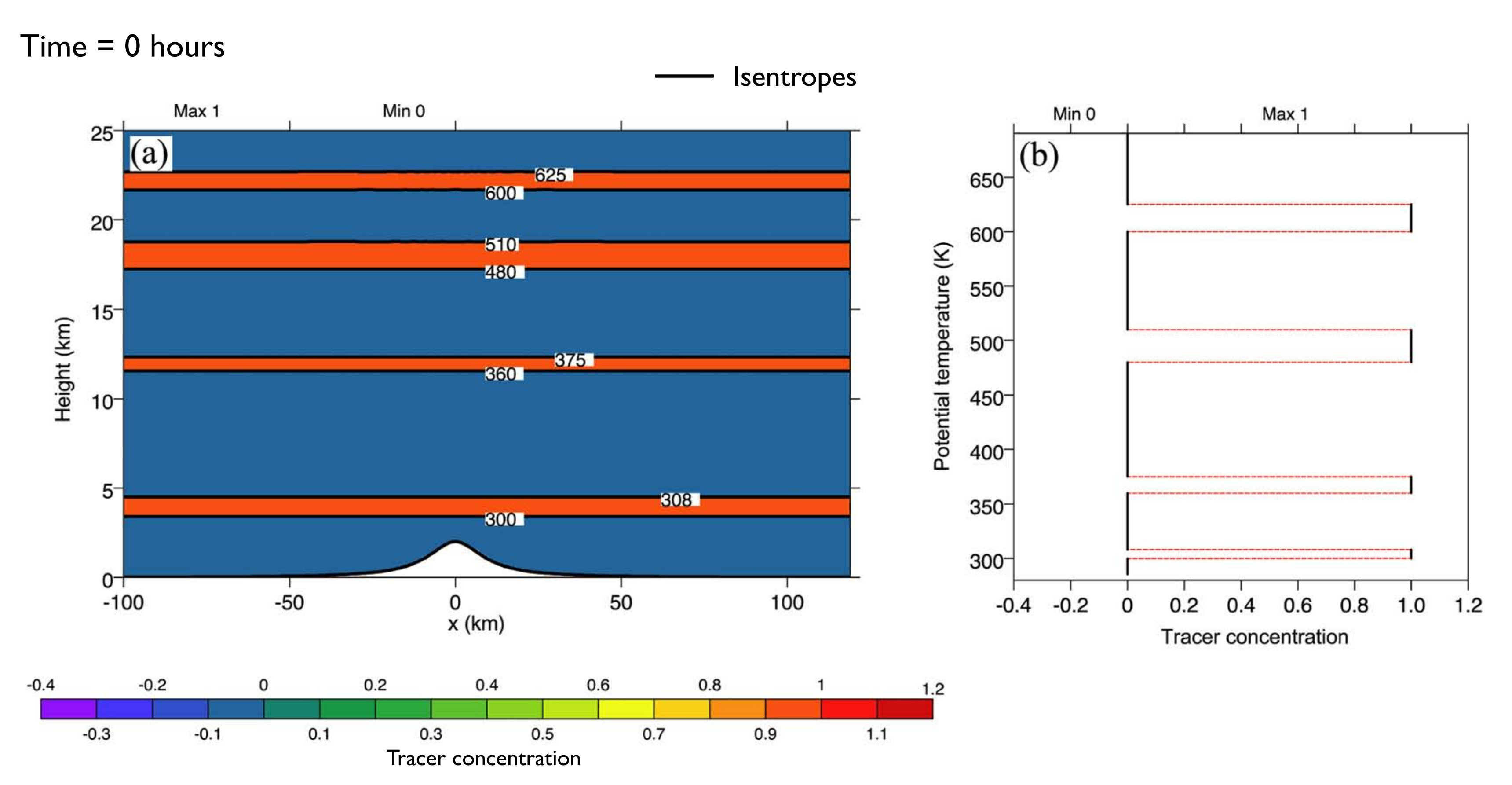
Time = 3 hours

Zonal wind component



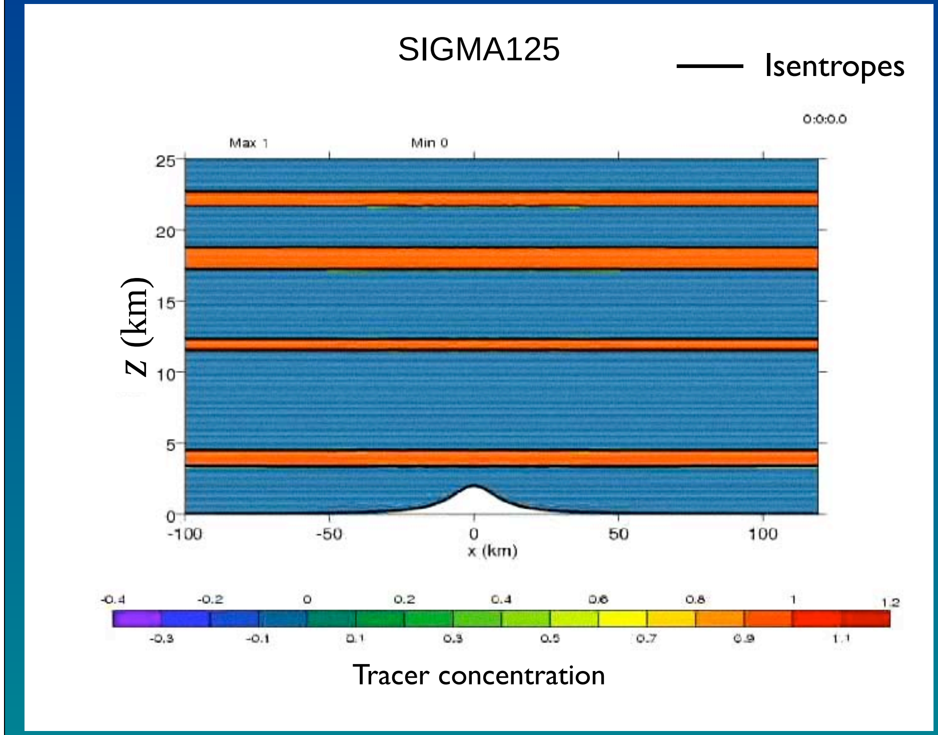
11 January 1972 Boulder windstorm simulations

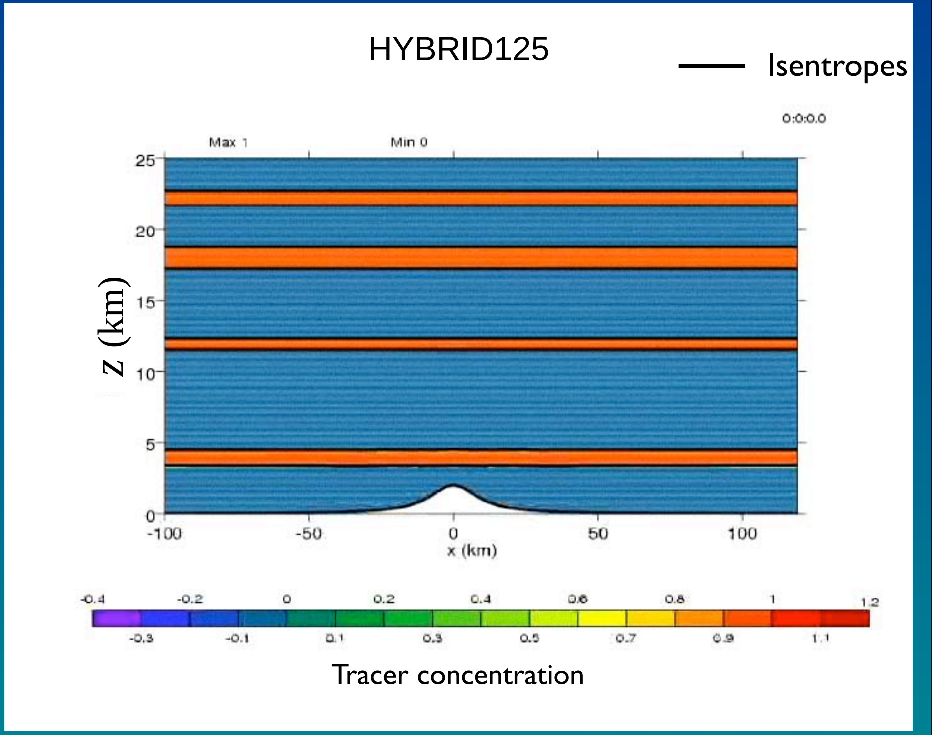
Tracer transport



11 January 1972 Boulder windstorm simulations

Tracer transport





11 January 1972 Boulder windstorm simulations

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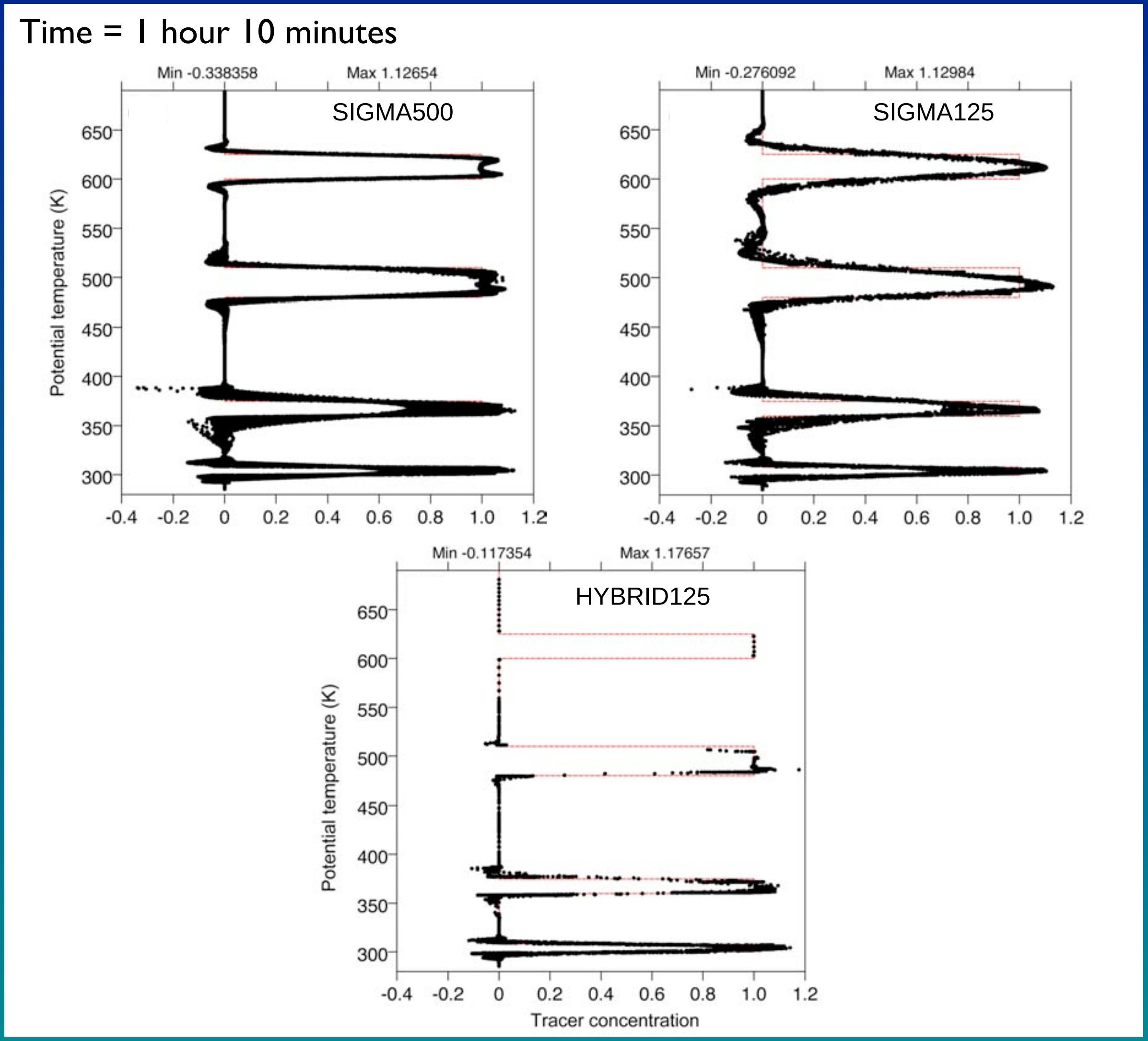
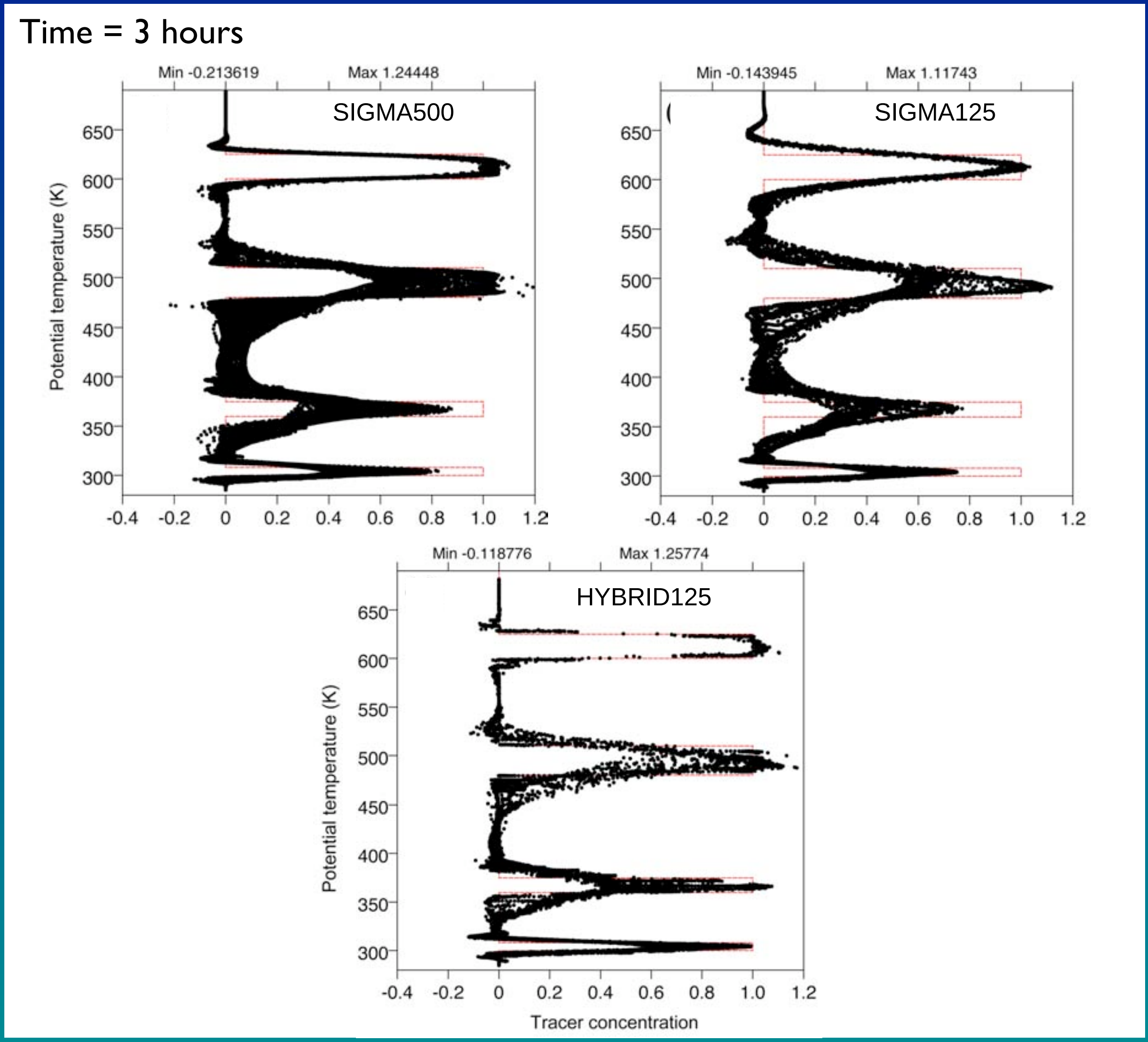


11 January 1972 Boulder windstorm simulations

Tracer transport



Conclusions

- The hybrid-coordinate method of KA97 can be adapted for fine-scale nonhydrostatic modeling
- Demonstrated advantages to using the QL coordinate
  - Less dispersion error in vertical transport
  - Improved resolution of features with high-static stability

Conclusions

- Wave-breaking turbulence partially suppressed with QL coordinate
- Verified QL representation of vertical momentum transport

Acknowledgements

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