PYRGEOMETER CALIBRATION UNCERTAINTY

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**Challenge**
- An uncertainty to meet the BSRN Longwave Target:
  - 5% or 10 W.m\(^{-2}\) (1997)

**Action**
- Used Guide to the Expression of Uncertainty in Measurement (GUM) methods to develop the uncertainty

**Result**
- Uncertainty < target
- Highlighted major contributors (focused developments)
CALIBRATION METHODS

Black Body

Comparison
PYRGEOMETERS

Eppley PIR

Kipp & Zonen CGR4
UNCERTAINTY DEVELOPMENT
Adapted from the GUM (Section 8)

1. Model (measureand) \( f(x_1, x_2, \ldots, x_n) \)
2. List inputs \( x_1, x_2, \ldots, x_n \)
3. input uncertainties \( u(X_n); \)
4. sensitivity coefficients \( \frac{\delta f}{\delta X_n}, \frac{\Delta f}{\Delta X_n} \)
5. Convert into measurand uncertainties; \( u(f(X_n)) = (\frac{\delta f}{\delta X_n}) \times u(X_n) \)
6. Combine uncertainties \( u(f) = \Sigma (f(X_n))^2 \)
7. Express as 95% uncertainty \( U(f) \).
MATHEMATICAL MODEL

**Calibration Equation**

\[ C = \frac{V}{E_{L,REF} + K_s(E_D - E_B) - E_B} \]

**Field Equation**

\[
\begin{align*}
E_L &= \frac{V}{C} + E_B - K.(E_D - E_B) \\
E_D &= \sigma.T_D^4
\end{align*}
\]

**PMOD Equation**

\[
E_{L,REF} = \frac{V_{REF}}{C_{REF}} \left(1 + K_{1,REF} \cdot \sigma \cdot \frac{T_{B,REF}^4}{T_{B,REF}}\right) + K_{2,REF} \cdot E_{B,REF} - K_{3,REF} \cdot (E_{D,REF} - E_{B,REF})
\]
$E = \sigma . T^4$

$T = \frac{1}{\alpha + \beta \cdot \text{Ln}(R) + \gamma \cdot (\text{Ln}(R))^3}$
SENSITIVITY COEFFICIENTS

1. Unit conversion factor
2. The sensitivity of the measure and to changes in the input

Determined by either:
- Partial derivative of the model w.r.t. inputs; or
- Calculating the impact of small changes by inputs.

\[ \frac{\delta f}{\delta X_n} \approx \frac{\Delta f}{\Delta X_n} \]

\[ u(f) = \sqrt{ \sum_i \left( \frac{\partial C_{IUT}}{\partial X_i} \cdot u(X_i) \right)^2 } \]
RELATIVE MAGNITUDE OF THE INPUTS

For a coefficient of 3.73 \( \mu V/W^{1}.m^{2} \) the uncertainty is 0.25 \( \mu V/W^{1}.m^{2} \) at a 95% confidence level.
For a coefficient of $12.3 \ \mu V.W^{-1}.m^2$ the uncertainty is $0.28 \ \mu V.W^{-1}.m^2$ at a 95% confidence level.
RETURN ON EFFORT
UNCERTAINTY IMPROVEMENTS
RELATIVE MAGNITUDE OF THE INPUTS

For a coefficient of 3.73 $\mu\text{V.W}^{-1}\text{.m}^2$ the uncertainty is $0.18\,\mu\text{V.W}^{-1}\text{.m}^2$ at a 95% confidence level.

\[ u(T) = 0.05^\circ\text{C} \]
For a coefficient of 12.3 \( \mu \text{V.W}^{-1}.m^2 \) the uncertainty is 0.26 \( \mu \text{V.W}^{-1}.m^2 \) at a 95% confidence interval.
TEST DATA EXAMPLE (PIR VS PIR)

Daily Files Calculated V.W-1.m²

± 1%
TEST DATA EXAMPLE (PIR VS CGR4)

PIR C vs Different Model References

± 1%
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
<th>Standard Uncertainty (u)</th>
<th>Sensitivity Coefficient (W.m²)</th>
<th>Degrees of Freedom (Dof)</th>
<th>u.s (W.m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>V&lt;sub&gt;REF&lt;/sub&gt;</td>
<td>-3.34E-04</td>
<td>V</td>
<td>1.78E-06</td>
<td>2.68E+05</td>
<td>12.5</td>
<td>0.48</td>
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<tr>
<td>R&lt;sub&gt;B,REF&lt;/sub&gt;</td>
<td>1.22E+04</td>
<td>Ω</td>
<td>5.5</td>
<td>-5.66E-02</td>
<td>12.5</td>
<td>-0.31</td>
</tr>
<tr>
<td>T&lt;sub&gt;B,REF&lt;/sub&gt;</td>
<td>293.27</td>
<td>K</td>
<td>0.050</td>
<td>-2.29E+01</td>
<td>26</td>
<td>-1.15</td>
</tr>
<tr>
<td>R&lt;sub&gt;D,REF&lt;/sub&gt;</td>
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<td>Ω</td>
<td>5.5</td>
<td>4.55E-02</td>
<td>12.5</td>
<td>0.25</td>
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<tr>
<td>T&lt;sub&gt;D,REF&lt;/sub&gt;</td>
<td>293.35</td>
<td>K</td>
<td>0.050</td>
<td>-2.29E+01</td>
<td>26</td>
<td>-1.15</td>
</tr>
<tr>
<td>C&lt;sub&gt;REF&lt;/sub&gt;</td>
<td>3.73E-06</td>
<td>V.W&lt;sup&gt;-1&lt;/sup&gt;.m&lt;sup&gt;-2&lt;/sup&gt;</td>
<td>1.26E-07</td>
<td>2.41E+07</td>
<td>92.77</td>
<td>3.04</td>
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<td>K&lt;sup&gt;-1&lt;/sup&gt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>K&lt;sub&gt;2, REF&lt;/sub&gt;</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>K&lt;sub&gt;3, REF&lt;/sub&gt;</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E&lt;sub&gt;L,REF,CAL&lt;/sub&gt;</td>
<td>328.03</td>
<td>W.m²</td>
<td></td>
<td></td>
<td>141.85</td>
<td>3.50</td>
</tr>
</tbody>
</table>
Measurement uncertainty less than (5% or 10 W.m⁻²)

3.50 W.m⁻² in 328.03 W.m⁻² (~1%)

Major contributors

Thermistors temperature measurement

Uncertainty Incomplete

Small but insignificant difference between CGR4 and PIR

WHAT HAVE WE ACHIEVED?
QUESTIONS
BUREAUS SKY COMPARISON TEST SYSTEM
REFERENCES


ATMOSPHERIC WINDOW

Public Domain,
https://commons.wikimedia.org/w/index.php?curid=34818020
SIGNIFICANCE OF DIFFERENCES

Significant

Not Significant

$X_a$ $X_b$ $U_{95}$
THERMOPILE

Irradiance W/m²

High Emissivity Surface $T_1$

Thermopile
Signal Volt = $S \times (T_1 - T_2)$

Heat Sink

Body $T_2$
**Overview Longwave Measurement**

Radiating to space

Radiating up

Atmosphere (255K)

Radiating down

Ground (288K)

**Table:**

<table>
<thead>
<tr>
<th>Source</th>
<th>Effective BB Temperature</th>
<th>Wavelength maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day (in)</td>
<td>5780 K (5507°C)</td>
<td>0.5 µm</td>
</tr>
<tr>
<td>Night (in)</td>
<td>255K (-18°C)</td>
<td>11µm</td>
</tr>
<tr>
<td>Terrestrial (out)</td>
<td>288K (15°C)</td>
<td>10µm</td>
</tr>
</tbody>
</table>

Recommended longwave uncertainty is: 5% or 10 W.m⁻² whichever is greater

**Diagram:**

- 0.4µm
- 0.7µm
- 3µm
- 100µm
TEST METHOD

1. Mount the test and the reference pyrgeometer (same model);
2. Select data from astronomically (-18°) dark skies;
3. Calculate reference irradiance (PMOD calibration equation);
4. Filter for significant nett irradiance data (>40 W.m⁻²);
5. Filter for stable irradiance data (<20 W.m⁻².h⁻¹);
6. Calculate pyrgeometer sensitivity coefficient (Field Equation);
7. Select nights (>4) with sensitivity coefficients with σ < 2%.
8. Calculate the uncertainty;
SENSITIVITY COEFFICIENTS (REFERENCE $E_L$)

\[ \frac{\partial E_{L,\text{REF}}}{\partial C_{\text{REF}}} = -\frac{V}{C_{\text{REF}}^2} (1 + K_1 \cdot \sigma \cdot T_{B,\text{REF}}^3) \]

\[ \frac{\partial E_{L,\text{REF}}}{\partial K_{1,\text{REF}}} = \frac{V}{C_{\text{REF}}} \sigma \cdot T_{B,\text{REF}}^3 \]

\[ \frac{\partial E_{L,\text{REF}}}{\partial K_{2,\text{REF}}} = -\sigma \cdot T_{B,\text{REF}}^4 \]

\[ \frac{\partial E_{L,\text{REF}}}{\partial K_{3,\text{REF}}} = \sigma \cdot (T_{B,\text{REF}}^4 - T_{D,\text{REF}}^4) \]

\[ \frac{\partial E_{L,\text{REF}}}{\partial V_{\text{REF}}} = \frac{(1 + K_{1,\text{REF}} \cdot \sigma \cdot T_{B,\text{REF}}^3)}{C_{\text{REF}}} \]

\[ \frac{\partial E_{L,\text{REF}}}{\partial T_{D,\text{REF}}} = -4 \cdot K_{3,\text{REF}} \cdot \sigma \cdot T_{D,\text{REF}}^3 \]

\[ \frac{\partial E_{L,\text{REF}}}{\partial T_{B,\text{REF}}} = 3 \cdot \frac{V_{\text{REF}}}{C_{\text{REF}}} \cdot K_{1,\text{REF}} \cdot \sigma \cdot T_{B,\text{REF}}^2 + 4 \cdot \sigma \cdot T_{B,\text{REF}}^3 \left( K_{2,\text{REF}} + K_{3,\text{REF}} \right) \]

\[ \frac{\partial E_{L,\text{REF}}}{\partial R_{B,\text{REF}}} = \frac{\partial E_{L,\text{REF}}}{\partial T_{B,\text{REF}}} \cdot \frac{\partial T_{B,\text{REF}}}{\partial R_{B,\text{REF}}} \]

\[ \frac{\partial T_{B,\text{REF}}}{\partial R_{B,\text{REF}}} = \frac{-\beta - 3 \cdot \gamma \cdot \text{Ln}(R_{B,\text{REF}})^2}{R_{B,\text{REF}} \left( \alpha + \beta \cdot \text{Ln}(R_{B,\text{REF}}) + \gamma \cdot \text{Ln}(R_{B,\text{REF}})^3 \right)^2} \]

\[ \frac{\partial E_{L,\text{REF}}}{\partial R_{D,\text{REF}}} = \frac{\partial E_{L,\text{REF}}}{\partial T_{D,\text{REF}}} \cdot \frac{\partial T_{D,\text{REF}}}{\partial R_{D,\text{REF}}} \]

\[ \frac{\partial T_{D,\text{REF}}}{\partial R_{D,\text{REF}}} = \frac{-\beta - 3 \cdot \gamma \cdot \text{Ln}(R_{D,\text{REF}})^2}{R_{D,\text{REF}} \left( \alpha + \beta \cdot \text{Ln}(R_{D,\text{REF}}) + \gamma \cdot \text{Ln}(R_{D,\text{REF}})^3 \right)^2} \]
\[ u(E_{L,\text{REF}})^2 = \left( \frac{\partial E_{L,\text{REF}}}{\partial C_{\text{REF}}} \cdot u(C_{\text{REF}}) \right)^2 + \left( \frac{\partial E_{L,\text{REF}}}{\partial K_{1,\text{REF}}} \cdot u(K_{1,\text{REF}}) \right)^2 \]
\[ + \left( \frac{\partial E_{L,\text{REF}}}{\partial K_{2,\text{REF}}} \cdot u(K_{2,\text{REF}}) \right)^2 + \left( \frac{\partial E_{L,\text{REF}}}{\partial K_{3,\text{REF}}} \cdot u(K_{3,\text{REF}}) \right)^2 \]
\[ + \left( \frac{\partial E_{L,\text{REF}}}{\partial V_{\text{REF}}} \cdot u(V_{\text{REF}}) \right)^2 + \left( \frac{\partial E_{L,\text{REF}}}{\partial T_{B,\text{REF}}} \cdot u(T_{B,\text{REF}}) \right)^2 + \left( \frac{\partial E_{L,\text{REF}}}{\partial R_{B,\text{REF}}} \cdot u(R_{B,\text{REF}}) \right)^2 \]
\[ + \left( \frac{\partial E_{L,\text{REF}}}{\partial T_{D,\text{REF}}} \cdot u(T_{D,\text{REF}}) \right)^2 + \left( \frac{\partial E_{L,\text{REF}}}{\partial R_{D,\text{REF}}} \cdot u(R_{D,\text{REF}}) \right)^2 \]
SENSITIVITY COEFFICIENTS (IUT)

\[
\frac{\partial C_{IUT}}{\partial E_{L,REF}} = -\frac{V}{E_{net}^2}
\]

\[
E_{net} = E_{L,REF} + K_s \cdot \sigma \cdot \left( T_{D,IUT}^4 - T_{B,IUT}^4 \right) - \sigma \cdot T_{B,IUT}^4
\]

\[
\frac{\partial C_{IUT}}{\partial V_{IUT}} = \frac{1}{E_{net}}
\]

\[
\frac{\partial C_{IUT}}{\partial R_{B,IUT}} = \frac{\partial C_{IUT}}{\partial T_{B,IUT}} \cdot \frac{\partial T_{B,IUT}}{\partial R_{B,IUT}}
\]

\[
\frac{\partial C_{IUT}}{\partial T_{B,IUT}} = \frac{V \cdot 4\cdot \sigma \cdot T_{B,IUT}^3 (K_s + 1)}{E_{net}^2}
\]

\[
\frac{\partial T_{B,IUT}}{\partial R_{B,IUT}} = \frac{-\beta - 3\cdot \gamma \cdot Ln(R_{B,IUT})^2}{R_{B,IUT} \left( \alpha + \beta \cdot Ln(R_{B,IUT}) + \gamma \cdot Ln(R_{B,IUT})^3 \right)^2}
\]

\[
\frac{\partial C_{IUT}}{\partial T_{D,IUT}} = \frac{V_{IUT} \cdot K_{s,IUT} \cdot 4\cdot \sigma \cdot T_{D,IUT}^3}{E_{net}^2}
\]

\[
\frac{\partial T_{D,IUT}}{\partial R_{D,IUT}} = \frac{-\beta - 3\cdot \gamma \cdot Ln(R_{D,IUT})^2}{R_{D,IUT} \left( \alpha + \beta \cdot Ln(R_{D,IUT}) + \gamma \cdot Ln(R_{D,IUT})^3 \right)^2}
\]

\[
\frac{\partial C_{IUT}}{\partial C_{IUT}} = 1
\]

\[
\frac{\partial C_{IUT}}{\partial C_{IUT}} = 1
\]
STEP 4. ATTRIBUTING STANDARD UNCERTAINTIES

- The two major evaluation categories are:
  - Type A - by statistical methods
  - Type B - by other means:

Purpose: to quantify a standard uncertainty
EXAMPLE: $U(R_B)$ UNCERTAINTY IN DMM MEASUREMENT OF BASE THERMISTOR RESISTANCE

YSI 44031 (-10°C to +55 °C) 50 kΩ to 5 kΩ

Type A: laboratory testing over a year

- Measure standard resistors with DMM in likely ambient conditions (climate chamber)
- From scatter of residuals determine
  - Standard uncertainty (SD) and DOF
EXAMPLE: \( U(R_B) \) UNCERTAINTY IN DMM MEASUREMENT OF BASE THERMISTOR

- YSI 44031 (-10°C to +55 °C) 50 kΩ to 5 kΩ

- Type B: Manufacture Specifications
  - The DMM specification
    - 100 ppm of reading + 10 ppm of range (95%)
      (over a 1 year calibration interval)
    - Expanded \( U=11\Omega \) for the range 50 kΩ to 5 kΩ (95%)
    - \( \text{Std} \ u = 5.5 \ \Omega \)
      (assuming \( cf \) 2.0 for a normal distribution)
INPUTS UNCERTAINTIES

\( u(EL, \text{REF}) \) Reference Irradiance, (C, K1, K2, K3 and PMOD Equation).

\( u(V\text{REF}) \) Ref thermopile voltage by the DMM.

\( u(TB, \text{REF}) \) Ref base temperature (manufacturing tolerance).

\( u(TD, \text{REF}) \) Ref dome temperature (manufacturing tolerance).

\( u(RB, \text{REF}) \) Ref base temperature (DMM resistance measurement).

\( u(RD, \text{REF}) \) Ref dome temperature (DMM resistance measurement).

\( u(V\text{IUT}) \) IUT thermopile voltage by the DMM.

\( u(TB, \text{IUT}) \) IUT base temperature (manufacturing tolerance).

\( u(TD, \text{IUT}) \) IUT dome temperature (manufacturing tolerances).

\( u(RB, \text{IUT}) \) IUT base temperature (DMM resistance measurement).

\( u(RD, \text{IUT}) \) IUT dome temperature (DMM resistance measurement).

\( u(\text{CRepeat}, \text{IUT}) \) Maximum ESDOM in C from a single night.

\( u(\text{CRepro}, \text{IUT}) \) ESDOM for combined nightly average.
STEP 2. INPUTS

Reference

1. Reference Irradiance $E_{L_{\text{ref}}}$
2. Reference calibration Coefficients ($C_{\text{ref}}, K_{1\text{ref}}, K_{2\text{ref}}, K_{3\text{ref}}$)
3. Temperatures base and dome ($T_{b_{\text{ref}}}, T_{d_{\text{ref}}}$)
4. Performance of the temperature sensors
5. Measurement of the sensors resistance
6. Measurement of differential sky to earth ($V_{\text{ref}}$)

Instrument under test

1. Temperatures base and dome ($T_{b_{\text{IUT}}}, T_{d_{\text{IUT}}}$)
2. Performance of the temperature sensors
3. Measurement of the sensors resistance
4. Measurement of differential sky to earth ($V_{\text{IUT}}$)
5. Stability of the comparison sky
6. Reproducibility of the comparison
TRACEABILITY

- PMOD to
  Bureau reference PIR and CGR4
- Bureau reference to
  Transfer Standards and new network instruments
- Transfer Standards to
  Network instruments

References under development